MA 225 B1 - CALC 3

For Exam 2, I recommend that you do as many problems as you can from the book (including review problems) for the sections we have covered in class. Below are a few problems you can practice with. The exam covers 17.7, all of chapter 18 (skipping 18.9), and 19.1.

(1) Is the function $f(x, y) = \sqrt{9 - x^2 - y^2}$ continuous as $(0, 0)$? Why?

(2) Let $f(x, y) = \frac{x^2-2}{y+2}$ and $P = (2, 0, 1)$.
   (a) find an equation for the line normal to the graph of $f(x, y)$ at P.
   (b) find an equation for the plane tangent to the graph of $f(x, y)$ at P.
   (c) find an equation for the line normal to the level set of $f(x, y)$ at $(x, y) = (2, 0)$.
   (d) find an equation for the line tangent to the level set of $f(x, y)$ at $(x, y) = (2, 0)$.

(3) Classify the critical points of the function $f(x, y) = x^2 + 2xy + y^3/6$.

(4) Classify the critical points of the function $f(x, y) = x^2 + y^4 - 2x - 4y^2 + 5$.

(5) Find the linear approximation formula for the function $f(x, y) = x^2y + y^2x$ around the point $(x_0, y_0) = (2, 1)$ and use it to give the approximation for $f(1.9, 1.2)$.

(6) The radius of the base of a cylinder at time 0 is 1 unit and is increasing at a rate of 0.5 units per second. At the same time the height is 2 units and is decreasing at a rate of 0.2 units per second. Find the rate of change of the volume at time 0. Is it increasing or decreasing?

(7) Let $f(x, y) = x^2y$; $x(s, t) = st$; $y(s, t) = s^2 - t^2$. Find $\frac{\partial f}{\partial s}$ and $\frac{\partial f}{\partial t}$.

(8) Find the gradient of $f(x, y) = x^2y^2 + xy + 2$ at $P = (1, 1)$.

(9) Find the point on the plane $2x + 3y + z - 14$ closest to the origin.

(10) Find the dimension of a rectangle, parallel to the axes, of maximal area, which can be inscribed in the ellipse $x^2/9 + y^2 = 1$.

(11) Find the direction of steepest *decrease* of the function $f(x, y, z) = x^2 + yz^2$ at the point $(1, 2, 3)$.

(12) Find the point on the hyperbola $xy = 1$ closest to the point $(1, 2)$.

(13) Calculate the iterated integral

$$\int_0^5 \int_0^2 x^2y \, dx \, dy$$

(14) Calculate the double integral

$$\iint_R x \sin(yx) \, dA$$

over the rectangle $0 \le x \le \pi/4$; $\quad 0 \le y \le 2$.

(15) Evaluate the following:

a) $$\int_0^1 \int_0^{\sqrt{1-y^2}} e^{x^2+y^2} \, dxdy$$

b) $$\int_1^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \frac{x^2}{x^2 + y^2} \, dydx$$

c) $$\int_0^1 \int_0^{\sqrt{1-y^2}} (x^2 + y^2)^{\frac{3}{2}} \, dxdy$$

d) $$\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_0^{y+2} xy \, dzdydx$$

e) $$\int_0^3 \int_{-\sqrt{9-x^2}}^0 \int_{\sqrt{x^2+y^2}}^{\sqrt{18-x^2-y^2}} e^{(x^2+y^2+z^2)^{\frac{3}{2}}} \, dzdydx$$

(16) Find the volume of the region lying inside both the cone $z^2 = x^2 + y^2$ and the sphere $x^2 + y^2 + z^2 = 2$.

(17) Find the volume of the region lying inside the cylinder $x^2+y^2 = 1$ and the sphere $x^2+y^2+z^2 = 4$.

(18) Check your answer from the previous problem using a different type of coordinates (e.g. if you used cylindrical, use cartesian coordinates).

(19) Find the volume of the solid lying between the spheres $x^2+y^2+z^2=4$ and $x^2+y^2+z^2=16$ and inside the cone $z^2=x^2+y^2$.

(20) Find the surface area of the graph given by $f(x,y)=127+y^2$ over the region

$$Q=\{(x,y)\mid 0\le x\le 2,\ 1\le y\le 2\}$$

(21) Find the surface area of the portion of the paraboloid $x=9-z^2-y^2$ lying over the yz–plane.

(22) Find the surface area of the part of the paraboloid $z=4-x^2-y^2$ lying inside the cylinder $x^2+y^2=1$.

(23) Find

$$\int\int_Q\int e^{2(x^2+y^2+z^2)^{\frac{3}{2}}}\,dV$$

where Q is the region bounded between the two half spheres

$$Q_1=\left\{(r,\theta,\phi)\mid 0\le r\le 2,\ 0\le\theta\le 2\pi,\ 0\le\phi\le\frac{\pi}{2}\right\}$$

and

$$Q_2=\left\{(r,\theta,\phi)\mid 0\le r\le 4,\ 0\le\theta\le 2\pi,\ 0\le\phi\le\frac{\pi}{2}\right\}$$

(24) Julie and Holly own a Calculus Tutoring Service. To advertise thier booming business, they have made a thin metal sign that is 4 yards wide and 2 yards high. Suppose its density, which is measured in pounds/yard^2, at each point is 3 times the distance from the center of the sign squared (hence $p(x,y,z)=3(x^2+y^2)$). In order to hang the sign, it must weigh less than 50 pounds. Will they be able to hang it up?

(25) Sebastian and Keith own a company that makes metal wedges. They make them in one size only which is given by the shape of the region

$$Q=\{(x,y,z)\mid -1\le x\le 1,\ 0\le y\le 3,\ 0\le z\le y\}$$

The density at the point (x,y,z) is given by $p(x,y,z)=1+x^2+y^2$ grams/cm^3 and the dimensions of all the wedges are all in centimeters. Suppose that they need to ship a wedge overnight to Seattle and teh shipping rate is \$30 per gram. Since neither of these guys is good at calculus, they ask you to figure out how much it will cost. What is the price?

(26) Evaluate

$$\int\int_R (xy^2+y)\,dA \qquad \text{on } R=\{0\le x\le 1,\ 0\le y\le 1\}$$

(27) Find the area of the region bounded by the graphs $y=x^2$ and $y=4$.

(28) Find the area of the region enclosed by the cardioid $r=1+\cos\theta$ in the first quadrant.

(29) Find the surface area of the region bounded by the cylinder $x^2+y^2=1$ inside the sphere $x^2+y^2+z^2=4$.

(30) Evaluate

$$\int_0^1\int_0^x\int_0^y e^y\,dzdydx$$

(31) Sketch the solid

$$Q=\left\{(x,y,z)\mid 0\le x\le 1,\ 0\le y\le\sqrt{1-x^2},\ 0\le z\le\sqrt{1-x^2-y^2}\right\}$$

Find the mass of Q is its density is

$$D(x,y,z)=x^2+y^2+z^2.$$

(32) Use a change of variables to find the area of the regien defined by

$$Q=\left\{(x,y)\,\middle|\,\begin{array}{ccccc}1&\le&x/y^4&\le&2\\2&\le&y/x^4&\le&3\end{array}\right\}$$

(33) Use a substitution to find the volume inside the ellipsiod

$$x^2 + y^2/4 + z^2/9 = 1$$

(34) Evaluate the work done by the force field $\mathbf{F} = xz\mathbf{i} + 2xy\mathbf{j} + z^2\mathbf{k}$ along the path given by $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$, in the time interval $0 \leq t \leq 2$.

(35) Evaluate the line integral

$$\int_C xy\, dx + y^2 dy$$

where C is the triangle with vertices $(0,0)$, $(0,1)$, $(1,0)$, in this order.

(36) Evaluate the line integral

$$\int_C x\cos(xy)\, dx + y\cos(xy)\, dy$$

along the circle C of radius 1 around the origin, going clockwise from $(0,1)$.