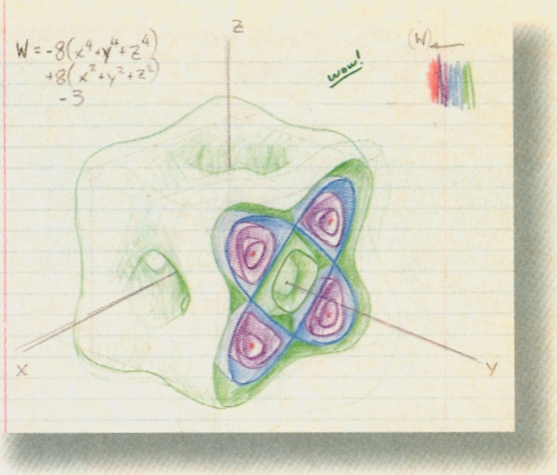
By Thomas Banchoff

The Best Homework Ever?



CASSIDY CURTIS'S MARVELOUS SURFACE DRAWINGS

Homework is a necessary chore for teachers as well as for students. Occasionally, though, a routine assignment produces something that is a pure joy.

Every instructor can recall students who have done outstanding jobs on assignments that stretch over a number of days, but I have a candidate for the Best Overnight Homework Ever. Nine years ago I was teaching an honors course in the calculus of several variables to a group of well-prepared first-year students. In my calculus courses I always encourage students to draw – first of all so they can sketch graphs of curves in the plane, and then so they can begin to visualize surfaces in space. Visualization techniques have always been important in mathematics and its applications, and they are especially so nowadays as sophisticated computer graphics enhance our ability to interpret phenomena we could not imagine a generation ago. But you can only really appreciate what the computer is showing you if you've tried to render the curves and surfaces freehand. Almost all of my students get the hang of it well enough to draw a pretty good surface, and some display a particular talent for illustrating mathematical ideas.

Right from the beginning, Cassidy Curtis '92 was unusually adept at drawing surfaces representing complicated algebraic expressions in two variables. He seemed instinctively able to choose just the right viewing angle to display the salient features of a surface, and he used color and shading to bring out key properties. He was equally impressive with colored pencils and with the interactive computer-graphics tools we were developing at Brown.

What astounded me, though, was his response to my first lecture on functions of three variables. We had spent a good deal of time analyzing contour lines of functions of two variables, such as the curves of equal temperature or pressure on a weather map or the contours of mountains on a topographical survey. I then introduced the analogous concept of contour surfaces for functions of three variables, such as the points of equal temperature in a room with a wood stove. I had a particular challenge in mind. The previous summer, I had attended a series of esoteric lectures at Berkeley given by Professor Friedrich Hirzebruch from the University of Bonn. Professor Hirzebruch was interested in a surface with many singularities (points where the surface looks like two cones with their points touching) defined using a polynomial in three complex variables:

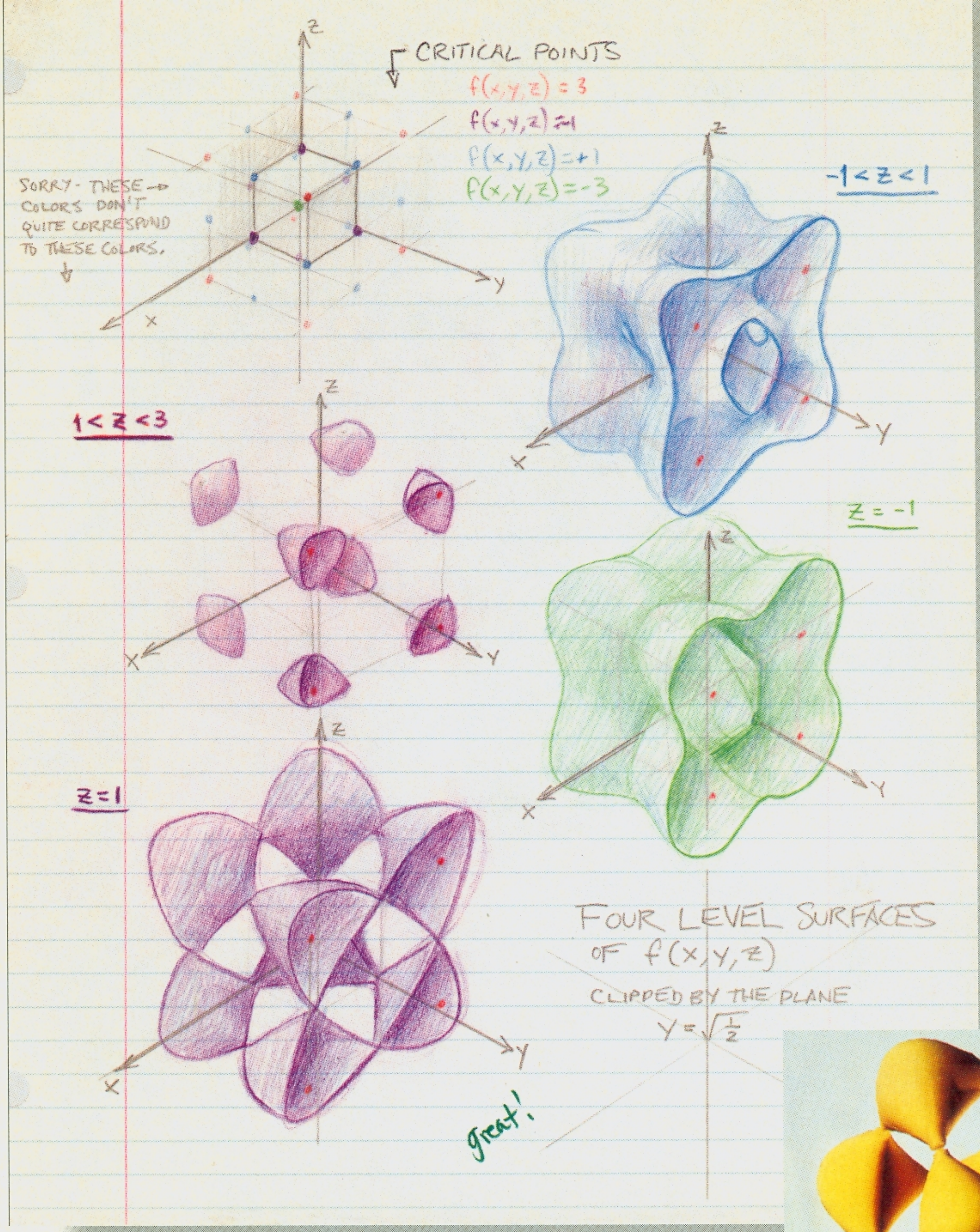
$$f(x,y,z)=$$
$$(-8x^4+8x^2-1)+(-8y^4+8y^2-1)+(-8z^4+8z^2-1)$$

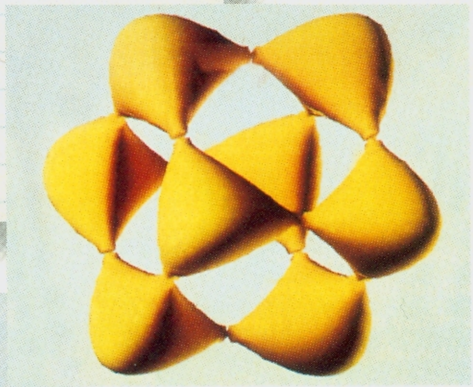
Although the lecturer said he knew a great deal about this function from the point of view of calculus and linear algebra, he regretted that he could not visualize its geometric shape. I thought our graphics team could help him out. I telephoned my sophomore assistant, Ed Chang '91, who rendered the surface on a computer using a contour-surface algorithm developed by Steve Ritter '85 and Kevin Pickhardt '85 in Professor Andries van Dam's computer-graphics course. Thanks to overnight mail and one-hour film developing, we had slides of the surface in Berkeley in time for Professor Hirzebruch's next lecture. He was delighted, and he has used our computer-graphics illustrations in his lectures and publications ever since.

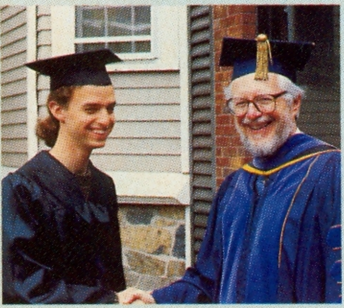
When I wrote that same equation on the board for my freshman honors students the following semester, I didn't tell them how difficult a visualization challenge it was. I had planned to spread the story over two weeks of class periods, climaxing by exhibiting our computer illustrations.

But I hadn't counted on Cassidy. The very next class, he came up to me and said he knew what those surfaces looked like. He showed me a page of drawings that were unlike anything I had ever seen from a student – perfect, hand-drawn renditions of the object we had created on the computer the previous summer with such labor, not just one image but an entire sequence. And on the next page of notebook paper, he showed how to stack all the color-coded surfaces together in four-space, something our computer could not do at the time!

Since then I have shown slides of Cassidy's work all over the world: in schools and universities, at conferences and art shows, for research mathematicians and for alumni groups. This is not only the best *freshman* math homework I have ever received. I contend it is the best overnight homework any teacher has ever received in any course at any level at any place in any subject at any time, ever, ever, ever. That is an extreme claim, but I'm still waiting for another teacher to produce a worthy challenger.







COURTESY THOMAS BANCHOFF

Thomas Banchoff (right, in photo) has been professor of mathematics at Brown since 1967. He is a pioneer in the geometry of the fourth and higher dimensions, and his Scientific American volume, *Beyond the Third Dimension,* recently came out in paperback. He told the story of his student's achievement during his acceptance speech for the Mathematical Association of America's national award for distinguished college or university teaching. After graduating in 1992, Banchoff's student, Cassidy Curtis (left, in photo), went on to do computer animation for XAOS, an advertising agency in San Francisco, where his credits included the countdown visual sequence for MTV. He then worked on animation for Pacific Data Images. Curtis is now in Seattle working on graphics projects with David Salesin '83, professor of computer science at the University of Washington.

Mathematics professor Tom Banchoff had never seen anything like the drawings (top) produced overnight by a freshman, Cassidy Curtis. On an accompanying sheet (opposite page) Curtis "stacked" his surface renditions, causing a teaching assistant to note: "Wow!" Above, a computer-generated illustration by Ed Chang '91 of the same algebraic function.