Geometry and Dynamics in Surfaces and 3-Manifolds

May 3-6, 2007 – Brown University

ABSTRACTS

- IAN AGOL, U. Illinois, Chicago

  The minimal volume orientable 2-cusped hyperbolic 3-manifolds

  We prove that the Whitehead and (-2,3,7)-pretzel link complements are the minimal volume orientable 2-cusped hyperbolic 3-manifolds, with volume 4K=3.66..., where K is Catalan's constant.

- MLADEN BESTVINA, Utah

  The cohomological dimension of Torelli groups

  We prove that the cohomological dimension of the Torelli group in genus $g > 1$ is $3g - 5$. This answers a question of Geoff Mess, who proved the lower bound and also settled the case $g = 2$. The technique involves decomposing Teichmüller space according to minimal length cycles representing a fixed homology class and studying the action of the Torelli group on the associated cell complex. This is joint work with Kai-Uwe Bux and Dan Margalit.

- DAVID DUMAS, Brown

  Slicing, skinning and grafting

  We show that a Bers slice of quasifuchsian space is never algebraic, meaning that its Zariski closure has strictly larger dimension. As an application we show that the skinning map of a hyperbolizable 3-manifold with boundary is never constant. The proof uses grafting and the theory of complex projective structures on surfaces. This is joint work with Richard Kent.

- ALEX ESKIN, U. Chicago

  Counting problems in Teichmüller space

  We apply ideas from the Ph.D. thesis of G.A. Margulis to Teichmüller space, and prove asymptotic formulas as R goes to infinity for for the number of closed geodesics of length at most R, the volume of ball of radius R, and the number of lattice points in a ball of radius R. Parts of this talk are joint work with Maryam Mirzakhani. Other parts are joint work with Jayadev Athreya, Sasha Bufetov and Maryam Mirzakhani.

- RICHARD KENT, Brown

  The beginning of the end

  The skinning map is a holomorphic self map of the Teichmüller space that arises naturally in Thurston's proof of Geometrization for Haken manifolds. A remarkable theorem of Thurston, the *Bounded Image Theorem*, says that the skinning map of a hyperbolic manifold with totally geodesic boundary has bounded image. Yair Minsky has asked if bounds on the diameter may be obtained given topological information about the manifold. I'll discuss "sharp" upper and lower bounds that only depend on the volume of the metric with totally geodesic boundary. These follow from: a filling theorem, which says that skinning maps converge uniformly as higher Dehn fillings are performed; the Bounded Image Theorem, together with a finiteness theorem of Jørgensen; and a theorem (joint with D. Dumas) that skinning maps are never constant.The theorem is sharp in the sense that the upper and lower bounds tend to infinity and zero, respectively, as the volume grows.

- CHRIS LEININGER, U. Illinois, Urbana-Champaign

  Convex cocompactness in the mapping class group

  Farb and Mosher defined a notion of convex cocompactness for subgroups of the mapping class group that mimics the notion of the same name for Kleinian groups. I'll explain why this class of groups is important, discuss a list of alternative characterizations analogous to those that exist for Kleinian groups, state some open questions in this area and explain some partial results. This is based on various pieces of joint work with R. Kent and S. Schleimer.

- HOWARD MASUR, U. Illinois, Chicago and Brown

  Quasi-geodesics in the pants complex

  We establish the extent to which coarse methods can be used to understand directly the behavoir of geodesics in the Weil-Petersson metric on Teichmüller space. By a Theorem of Brock, the Weil-Petersson metric is quasi-isometric to the pants complex, so geodesics become quasi-geodesics in the pants complex. We show that in genus 2 (and cases of equivalent complexity) the pants complex is strongly relatively hyperbolic with respect to product regions corresponding to pants decompositions containing separating curves (hyperbolicity was shown in lower complexity by Brock and Farb). This relative hyperbolicity provides for control on behavior of quasi-geodesics, hence Weil-Petersson geodesics. We show no such control is possible in any higher complexity cases. This represents joint work with Jeff Brock.

- HOSSEIN NAMAZI, Princeton

  Heegaard splittings and hyperbolic geometry

  An important question regarding geometrical structure of a 3-manifold is to give a description of its shape by starting from a combinatorial data. While proving Thurston's ending lamination conjecture, Brock-Canary-Minsky provided such a coarse description for the geometry of incompressible ends of a hyperbolic 3-manifold. We try to adopt this approach to the more general case when the boundary is possibly compressible. We also show how this description together with the combinatorics of an appropriate Heegaard splitting leads to a description of the hyperbolic metric on a closed 3-manifold.

- KASRA RAFI, U. Connecticut

  On distinguishing curve complexes

  We show that, if the curve complexes of a pair of surfaces are quasi-isometric and each has connected Gromov boundary then the two surfaces have equal complexity. This is joint work with Saul Schleimer.

- ALAN REID, U. Texas, Austin

  Heegaard genus and Property τ for hyperbolic 3-manifolds

  This talk will discuss the proof of:

  Theorem: *Let M be a finite volume hyperbolic 3-manifold. Then M has a co-final family of finite sheeted covers for which the infimal Heegaard gradient is positive.*

  The proof uses work of Lackenby that relates positivity of infimal Heegaard gradient to Property τ. The main part of the argument is to show:

  Theorem: *Let Γ be a finitely generated non-elementary Kleinian group. Then Γ has a co-final family of finite index normal subgroups $\mathscr{L} = \{N_i\}$ with respect to which Γ has Property τ.*

- RICHARD SCHWARTZ, Brown

  Outer billiards, unbounded orbits, and the modular group

  Outer billiards is a simple dynamical system based on a convex shape. This system was introduced by B.H. Neumann in the 1950s and popularized by J. Moser in the 1970s as a toy model for celestial mechanics. All along, the central question has been: can there exist an outer billiards system with unbounded orbits? In my talk I will explain my recent solution of this problem, in which I prove that outer billiards on the so-called *Penrose kite* has unbounded orbits. I will also discuss the vague analogy between outer billiards and hyperbolic 3-manifolds that partly motivated my solution, and also a precise connection between outer billiards and the modular group. The connection to the modular group -

something I have yet to rigorously prove - implies that there are uncountably many kites on which outer billiards has unbounded orbits and thereby gives a second solution of the Neumann-Moser problem.

- JUAN SOUTO, U. Chicago

  Rank of the fundamental group and hyperbolic geometry

  I will describe some result relating the geometry and topology of hyperbolic 3-manifolds to the number of elements needed to generate of their fundamental group.

- PETER STORM, Stanford

  A deformation of a hyperbolic 4-orbifold

  There is a finitely generated subgroup of reflections in the walls of the hyperbolic 24-cell which has a one dimensional representation space. This deformation space has some qualitative properties reminiscent of 3-dimensional hyperbolic Dehn surgeries. This work is joint with Steven Kerckhoff.