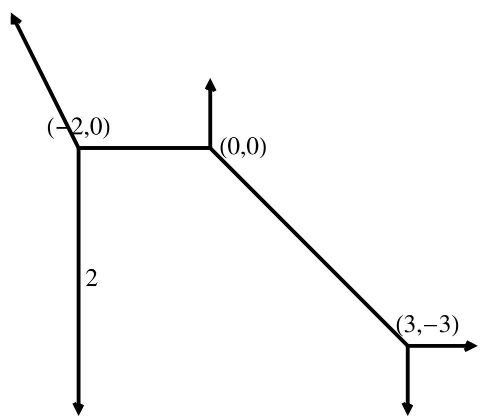
MATH 285y TROPICAL GEOMETRY SPRING 2013
PROBLEM SET 1, DUE THURSDAY FEBRUARY 14

1. Draw the tropical hypersurfaces for each of the following Laurent polynomials f over the field $\mathbb{C}\{\{t\}\}$.
   (a) $f = t^3y^3 + y^2 + xy^2 + y + t^{-1}xy + x^2y + t^3 + x + x^2 + t^2x^3$
   (b) $f = xy + 5xy^2 - xy^3 + tx^2y - t^2x^2y^2 - 3t^2x^3y$
   (c) $f = t + xy + x^{-1}y + xy^{-1} + x^{-1}y^{-1}$
   (d) $f = 1 + 2x + 3y + 4z$
   (e) $f = tx + y + z$
2. Find a polynomial $f \in \mathbb{C}\{\{t\}\}[x, y]$ giving rise to the tropical plane curve below. Here, the edge multiplicities are 1 unless otherwise noted; the upper left ray has direction $(-1, 2)$.



3. (Exercise 2.7.4 in book) Show that if K is an algebraically closed field with valuation val : $K^* \to \mathbb{R}$, then its residue field $k = R/m$ is also algebraically closed.
4. Prove the *balancing condition* for tropical plane curves: for any Laurent polynomial $f \in K[x^{\pm}, y^{\pm}]$, for each vertex $v \in \mathrm{TropV}(f)$, we have the following zero-tension condition on the edges at each vertex:
$$\sum_{e \ni v} \mathrm{mult}(e) \cdot e' = 0$$
where the e' is the vector of lattice length 1 in the direction of e. Check this condition for each of the examples (a)-(c) above.

5. Prove tropical Bézout's theorem for transverse intersections: two plane tropical curves C and D of degrees c and d with finite intersection meet in $c \cdot d$ points, where each point p is counted with multiplicity

$$\operatorname{mult}(e)\operatorname{mult}(f)\left|\det\begin{pmatrix} u_1 & u_2 \\ v_1 & v_2 \end{pmatrix}\right|$$

Here e and f are the edges of C and D containing p, and $e' = (u_1, v_1)$ and $f' = (u_2, v_2)$ are vectors of lattice length 1 in the directions of e and f, respectively.

One possible hint: consider $C \cup D$ itself as a tropical plane curve.

6. (Exercise 2.7.3 in book) Pick two triangles P and Q that lie in non-parallel planes in $\mathbb{R}^3$. Draw their Minkowski sum $P + Q$ and its normal fan. How many faces of each dimension does $P + Q$ have? Verify that the normal fan of $P + Q$ is the *common refinement* (p.75) of the normal fans of P and Q.