MATH 2110, MANIFOLDS

Instructor: Tom Goodwillie

K Hour (Tuesday and Thursdays 2:30-3:50) in Kassar 105

Office hour Tuesdays at 10:30, Thursdays at 1:00, or by appointment

The “textbook” will be a set of notes by Bjorn Dundas, available at http://folk.uib.no/nmabd/dt/dtcurrent.pdf.
This text does not actually cover all of the topics that we will study, but I like the point of view. I plan to supplement it by writing some notes of my own about various things, such as Lie groups, Stokes’s Theorem, and the Frobenius Integrability Theorem. Of course students might want to look at other sources. There are many books on the subject of smooth manifolds, for example Warner’s “Foundations of Differentiable Manifolds and Lie Groups” or Lee’s “Introduction to Smooth Manifolds”.

A few exercises will be assigned every week. The final exam will be a take-home. Grades will be based equally on homework and final.

Here is an outline of topics to be covered (though the order of the topics is not certain).

1. What is a smooth manifold? Definition in terms of charts and atlases. Submanifolds.

2. Tangent and cotangent vectors. The (co)tangent vector space of M at a point. Tangent and cotangent fields on M. The manifold of all (co)tangent vectors to M.

3. Transversality. As a consequence of the Inverse Function Theorem it is often easy to recognize when a subset of a manifold is a submanifold. For example, when two submanifolds intersect transversely then their intersection is a submanifold.

3. Partitions of unity. This is a tool for patching together local data to create global structures.

4. General position. Sard’s theorem and some applications. For example, submanifolds “usually” intersect transversely.

5. Tangent vector fields. The Lie bracket operation. The fact that ODEs with initial data have unique local solutions means that a tangent field determines a (local) flow.

6. Lie groups, a very brief introduction with examples. A Lie group is a manifold with a (compatible) group structure.

7. Vector bundles. The first examples are the tangent and cotangent bundle of a manifold. (Co)tangent fields are sections of the (co)tangent bundle. Vector bundles can be combined with each other using operations of (multi)linear algebra.

8. Differential forms. These are sections of exterior powers (antisymmetrized tensor powers) of the cotangent bundle. They are used in Stokes’s theorem, a global n-dimensional version of the Fundamental Theorem of Calculus.

9. The Frobenius integrability theorem.