1 Due Tuesday 2/9/2010

1. Show that $\mathbb{RP}^n$ is compact, Hausdorff, and second countable, thus completing the proof that it is a smooth manifold. (This is exercises 1.2 and 1.3 on page 8 of Lee.)

2. Problem 1-7 on page 29: Construct the smooth manifold $\mathbb{C}P^n$.


4. Suppose that $0 < a < b < c$. Verify that the ellipsoid $E$ defined by $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ is a smooth manifold in $\mathbb{R}^3$. For which values of $R > 0$ is $E$ transverse to the sphere $x^2 + y^2 + z^2 = R^2$?

2 Due Thursday 2/25/2010

5. Problem 2-1 on page 57. (By the way, the map in part (c) is such that $F(z, w)$ depends only on the point $[z : w]$ in $\mathbb{C}P^1$.)

6. Problem 2-4 on page 58.

7. Problem 2-6 on page 58.


Show that every closed subset of a topological manifold $N$ is $f^{-1}(0)$ for some continuous function $f : N → \mathbb{R}$. If $N$ is smooth, show that every closed set is an intersection of sets $f^{-1}(0)$ with $f$ smooth. Conclude that if $M$ and $N$ are topological [resp. smooth] manifolds and $F : M → N$ is a map of point sets such that for every continuous [resp. smooth] function $f : N → \mathbb{R}$ the function $f \circ F$ is continuous, then $F$ is continuous. (This yields a strengthening of the result proved in Problem 6 on the last assignment (Lee’s 2-4).)

Suppose that $F : M → N$ is smooth map. Show that if the map $F^* : C^\infty(N) → C^\infty(M)$ defined by $F^* f = f \circ F$ is bijective then $F$ is a homeomorphism (and therefore by Problem 7 of the last assignment a diffeomorphism). In fact show that if $F^*$ is surjective then $F(M)$ is closed in $N$ and $F : M → F(M)$ is a homeomorphism; and that if $F^*$ is injective then $F(M)$ is dense in $N$. [You may need the existence of a proper smooth map $M → [0, \infty)$ – see Prop. 2.28 of Lee.]

Problem 3-6 on page 79.

Problem 4-6 on page 101.

Problem 4-13 on page 102. (Is associativity relevant?)

Problem 4-14 on page 102.

Problem 4-18 on page 102.

Due Tuesday 3/23/2010

Problem 5-8 on page 122.

Problem 5-9 on page 122.

Problem 6-5 on page 151.

Problem 6-8 on page 152.

Problem 6-13 on page 153.

Problem 6-14 on page 154.