Big q-Ample Line Bundles

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A recent paper of Totaro (1007.3955) develops a theory of q-ample bundles in characteristic 0. Specifically, a line bundle $\mathcal{L}$ on X is q-ample if for every coherent sheaf $\mathcal{F}$ on X, there exists an integer m_0 such that $m \geq m_0$ implies $H^i(X, \mathcal{F} \otimes \mathcal{L}^m) = 0$ for $i > q$. We show that if $\mathcal{L}$ is a big line bundle on a complex projective variety X which fails to be q-ample, then there exists a codimension 1 subscheme D of X such that the restriction of $\mathcal{L}$ to D is not q-ample.