Let me fix some notation. I will denote a KM stack in our sense, as in the flows document, by (F, N_σ, N) - so F is a fan in a lattice N, and N_σ is a submonoid of N for each cone σ in F, with the appropriate compatibility conditions. I will denote their stacks by $(F', L), \sigma : L \to N$, where F' is a fan in L (different notation for the lattice that contains F!) and σ is the finite cokernel morhpism. Recall the geometric realization process in the two cases: For the KM stack, one takes the toric variety defined by the monoid N_σ for the maximal cones, then divides by the kernel of the map of tori $T(N_\sigma^{\mathrm{gp}})$, and glues by the compatibility condition. The geometric realization of $(F', L \to N)$ is $[X(F')/Ker(T(L) \to T(N)]$. The proposed correspondence is

Proposition: To go from a KM fan (F, N_σ, N) to a GS fan, form L by taking the colimit of the lattices $(N_\sigma)^{\mathrm{gp}}$, where the arrows in the diagram are the ones given from the fan: $\tau < \sigma \Rightarrow N_\tau \to N_\sigma$. This will be the lattice L in GS. The fan F' is by taking the colimit of the cones σ but as subcones of $L_\mathbb{R}$, according to the inclusion $\sigma \subset (N_\sigma)^{\mathrm{gp}}_\mathbb{R} \to L_\mathbb{R}$. (So you break the fan into its pieces and put it together in the bigger lattice according to the same gluing maps). The lattice N is the same for both GS/KM, and the map $L \to N$ is the one coming from the universal property of the colimit.

Proof: By the universal property of the colimit, there is a map between geometric realizations $X(F, N_\sigma, N) \to X(F', L \to N)$. To show it is an isomorphism it suffices to check it is an isomorphism locally. So let σ be one cone of F and consider the restrictions $X(\sigma, N_\sigma, N) \to X(\sigma, L, N)$. The result follows from Theorem B.3. in the paper [GS1].

Example 1: Let (F, N_σ, N) be the cone of $\mathbb{P}(1, 2)$, with $N = \mathbb{Z}$, $F = \{0, \sigma = \langle e_1 \rangle, \tau = \langle -e_1 \rangle\}$, $N_\sigma = 2\mathbb{N}$, $N_\tau = -\mathbb{N}$. To get the GS lattice L, we form the colimit, which is $\mathbb{Z} \oplus \mathbb{Z}$. Inside it, we have the two cones σ, τ, glued along 0, but now σ is along the first coordinate and τ is along the second. So the fan F' is $(x, 0) \cup (0, y), x, y \geq 0$. The map $L \to N$ is the map $\mathbb{Z}^2 \to \mathbb{Z}$ given by the matrix $(2, 1)$. This gives a toric variety $X(F') = \mathbb{A}^2 - 0$, map of tori $(\mathbb{C}^*)^2 \to \mathbb{C}^*, (s, t) \mapsto s^2 t$, with kernel $(s, s^(- 2)) \cong \mathbb{C}^*$, and the GS stack is $[X(F')/\mathbb{C}^*] = \mathbb{P}(1, 2)$.

Example 2: Let $N = \mathbb{Z}^2$, F the fan given by the cones

$$\{0, \tau = \langle (1, 2) \rangle, \sigma_1 = \langle (1, 0), (1, 2) \rangle, \sigma_2 = \langle (1, 2), (0, 1) \rangle\}$$

$N_{\sigma_1} = \mathbb{N}^2 = \mathbb{N}(1, 0) \oplus \mathbb{N}(1, 2)$, $N_{\sigma_2} = \mathbb{N}(1, 2) \oplus \mathbb{N}(0, 1) = N$, $N_\tau = \mathbb{N}(1, 2)$.

The lattice L is the coproducct $\mathbb{Z}^2 \oplus_\mathbb{Z} \mathbb{Z}^2$, where the first map is $\mathbb{Z} \to \mathbb{Z}^2, 1 \mapsto e_2$, the second $1 \mapsto e_1$. So all in all, $L = \mathbb{Z}^3$. The fan F' in $\mathbb{Z}^3$ is given by $\langle e_1, e_2 \rangle$ and

$\langle e_2, e_3\rangle$. The toric variety it determines is $X(F') = \mathbb{A}^3 - \mathbb{A}^1(0,1,0)$ (it is obtained from the fan of $\mathbb{A}^3$ by removing the interior of the first octant and the plane $\angle e_1, e_3\rangle$, so $X(F')$ is obtained from $\mathbb{A}^3$ by removing the orbits corresponding to these cones). The map $\mathbb{Z}^3 \to N = \mathbb{Z}^2$ is given by the matrix

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix}$$

which gives a map of tori $(\mathbb{C}^*)^3 \to (\mathbb{C}^*)^2, (x,y,z) \mapsto (xy, y^2z)$. Its kernel is $(y^{-1}, y, y^{-2}), y \neq 0$. So the GS stack is

$$[(\mathbb{A}^3 - \mathbb{A}^1(0,1,0))/\mathbb{C}^*], t(x,y,z) = (t^{-1}x, ty, t^{-2}z)$$

We check this agrees with the geometric realization $X(F, N_\sigma, N)$ on charts: on the chart $z \neq 0$, we might as well take $z = 1$, so $t = \pm 1$, and the quotient is $\mathbb{A}^2/\mathbb{Z}_2$, $t(x,y) = (-x,-y)$. This is precisely the chart on (F, N_σ, N) given by σ_1: there we have $X(N_{\sigma_1}) = \mathbb{A}^2$, and map of tori $(\mathbb{C}^*)^2 \to (\mathbb{C}^*)^2$ given by $(x,y) \mapsto (xy, y^2)$, with kernel $\mathbb{Z}_2 = \{(1,1), (-1,-1)\}$. For the chart $x \neq 0$ we have that the quotient is $\mathring{A}^2$, which agrees with $X(F, N_\sigma, N)$ trivially.