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Dan,

Yes, I agree that the functor

$$\begin{array}{rcl} \mathbf{LogSch}^{\mathrm{op}} & \to & \mathbf{Sets} \\ X^\dagger & \mapsto & \mathrm{Hom}_{\mathbf{LogSch}}(X^\dagger \times k^\dagger, k^\dagger) \end{array}$$

(here $k^\dagger := \mathrm{Spec}(0 : \mathbb{N} \to k)$ and the product is the one in log schemes over k) is represented by

$$\mathrm{Spec}\, k \coprod \coprod_{d>0} \mathrm{Spec}(x : \mathbb{N} \to k[x]).$$

Indeed, if $X^\dagger = (X, \mathcal{M}_X)$, then $\mathrm{Hom}_{\mathbf{LogSch}}(X^\dagger \times k^\dagger, k^\dagger)$ is just the set of commutative diagrams

$$\begin{tikzcd} \mathcal{M}_X(X) \oplus \underline{\mathbb{N}}(X) \arrow[r, "(\alpha,0)"] & \Gamma(X, \mathscr{O}_X) \\ \mathbb{N} \arrow[u, "h"] \arrow[r, "0"] & k \arrow[u] \end{tikzcd}$$

and this certainly splits as a disjoint union over the possible $d := \pi_2 h(1) \in \underline{\mathbb{N}}(X)$. When $d = 0$, commutativity forces $m := \pi_1 h(1) \in \mathcal{M}_X(X)$ to be such that $\alpha(m) = 0 \in \Gamma(X, \mathscr{O}_X)$ and the map is the pullback of $\mathrm{Id} \in \mathrm{Hom}_{\mathbf{LogSch}}(k \times k^\dagger, k^\dagger)$. After passing to a component of X where $d \in \mathbb{N}_{>0}$ is constant, the map in question fits into a diagram

$$\begin{tikzcd} \mathbb{N} \oplus \mathbb{N} \arrow[r, "(x,0)"] \arrow[d, "m \oplus 1"] & k[x] \arrow[d, "x \mapsto \alpha(m)"] \\ \mathcal{M}_X(X) \oplus \mathbb{N} \arrow[r, "(\alpha,0)"] & \Gamma(X, \mathscr{O}_X) \\ \mathbb{N} \arrow[u, "h=(m,d)"] \arrow[r, "0"] \arrow[uu, bend left=60, "(1,d)"] & k \arrow[u] \arrow[uu, bend right=60] \end{tikzcd}$$

so is pulled back from the universal map given by (Spec(_ $\to$ _) of) the "big" square.

–Danny