# MATH 251 PROBLEMS - LOCALIZATION AND LIMITS

(1) Localization:
   (a) Lang p 115 ex 1,3,4
   (b) Lang p 168 ex 10a,b
   (c) If $R$ is a domain and $0 \notin S \subset R$ a multiplicative set, show that $S^{-1}R$ is a subset of the field of fractions of $R$.
   (d) If $S \subset R$ contains no zero divisors, show that $R \to S^{-1}R$ is injective.
   (e) If $R = A \times B$ and $S = A \times \{1\}$, show that $S^{-1}R = B$.
   (f) Let $S \subset R$ be multilicative and $T = \{su | s \in S, u \in R^{\times}\}$. Thow that $S^{-1}R \simeq T^{-1}R$.
   (g) Prove that if $p \neq q$ are distinct primes, then $\mathbb{Z}_{(p)} \not\simeq \mathbb{Z}_{(q)}$.
   (h) Let $M$ be a finitely generated $R$ module and $S \subset R$ multiplicative. Show that $S^{-1}M = 0$ if and only if tehre is $d \in S$ with $dM = 0$.

(2) Limits:
   (a) Give an example of a directed system of modules $(M_i, \phi_{ij})$ with $M_i \neq 0$ for all $i$ but $\varinjlim M_i = 0$.
   (b) Define a homomorphism of directed systems $h : (M_i, \phi_{ij}) \to (N_i, \psi_{ij})$ with the same index set $I$ to be a collection of homomorphisms $h_i : M_i \to N_i$ such that $h_j \circ \phi_{ij} = \psi_{ij} \circ h_i$ (draw the diagrams). Show that this is a category $R - mod_I$, and the limit $\varprojlim : R - mod_I \to R - mod$ is a functor (define it n arrows!).
   (c) Lang p. 169-171 ex 17,19,21,23