(1) Let n be an integer and s the sum of its digits when written in base p. Show that $v_p(n!) = (n-s)/(p-1)$.

(2) Let $u \in U_1 = 1 + p\mathbb{Z}_p$ and $\alpha \in \mathbb{Z}_p$. Show that

$$u^\alpha = \lim_{a \to \alpha,\ a \in \mathbb{Z}} u^a$$

exists in U_1.

(3) What are the absolute values of $\mathbb{Q}(i)$ extending the standard ones of $\mathbb{Q}$?

(4) The maximal unramified extension of $\mathbb{Q}_p$ is obtained by adjoining all roots of 1 of order prime to p.

(5) The maximal tamely ramified *abelian* extension of $\mathbb{Q}_p$ is finite over the maximal unramified extension of $\mathbb{Q}_p$.

(6) Show that the maximal unramified extension of $\mathbb{F}_p((t))$ is $\cup_n \mathbb{F}_{p^n}((t))$. Show that the maximal tamely ramified extension is $\cup_{n, p \nmid m} \mathbb{F}_{p^n}((t^{1/m}))$.

(7) Samuel IV.3B. Part (c) refers to the fact that $X^3 + 10X + 1$ is irreducible since it has no integer roots, and that the ring of integers is generated by a root x since $|D(x)|$ is a prime.

(8) Group exercise: Samuel V.7B.

(9) Group exercise: Samuel review exercises I.

(10) Group exercise: Samuel review exercises III.