Description of admissibility

Definition

J is $\mathcal{I}$-admissible if $J \leq v(\mathcal{I})$.

Lemma

This is equivalent to $\mathcal{I}\mathcal{O}_{Y'} = E^\ell \mathcal{I}'$, with $J = \bar{J}^\ell$ and $\mathcal{I}'$ an ideal.

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- Write $J = (x_1^{a_1}, \ldots, x_k^{a_k})$ and $\mathcal{I} = (f_1, \ldots, f_m)$.
- Expand $f_i = \sum c_\alpha x_1^{\alpha_1} \cdots x_n^{\alpha_1}$.
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- $\Leftrightarrow \quad \sum \frac{\alpha_j}{a_j} \geq 1$ for all i and α such that $c_\alpha \neq 0$.

Consequences

- J is $\mathcal{I}_1, \mathcal{I}_2$-admissible $\Rightarrow$ J is $\mathcal{I}_1 + \mathcal{I}_2$-admissible.
- J is $\mathcal{I}$-admissible $\Rightarrow$ J^a is $\mathcal{I}^a$-admissible.
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Proof.

$$v_J\left(\frac{\partial x^\alpha}{\partial x_j}\right) = \sum \frac{\alpha_i}{a_i} - \frac{1}{a_j} \geq 1 - \frac{1}{a_1}.$$

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Combining:

Proposition

A center J is $\mathcal{I}$-admissible if and only if $J^{(a_1-1)!}$ is $C(\mathcal{I}, a_1)$-admissible.