Properties of the invariant

Theorem (MC-invariance [Włodarczyk, Kollár])

Given maximal contacts x_1, x_1' *and a common extension to regular systems of parameters* $(x_1, x_2, \ldots, x_n)$ *and* $(x_1', x_2, \ldots, x_n)$, *there are étale* $\pi, \pi' : \tilde{Y} \rightrightarrows Y$ *such that*

$$\pi^* x_1 = \pi'^* x_1'$$
$$\pi^* x_2 = \pi'^* x_2$$
$$\vdots \quad \vdots$$
$$\pi^* C(\mathcal{I}, a_1) = \pi'^* C(\mathcal{I}, a_1).$$

Proposition

inv_p *is functorial, well-defined, upper-semi-continuous.*

inv_p is functorial, well-defined, upper-semi-continuous.

- $a_1 = \text{ord}$, $\mathcal{D}(\mathcal{I})$ are functorial for smooth maps $\Rightarrow C(\mathcal{I}, a_1)$ is functorial,

inv_p is functorial, well-defined, upper-semi-continuous.

- $a_1 = \mathrm{ord}$, $\mathcal{D}(\mathcal{I})$ are functorial for smooth maps $\Rightarrow C(\mathcal{I}, a_1)$ is functorial,
- so by induction $\mathrm{inv}_p(\mathcal{I}, x_1, \ldots, x_n)$ is functorial.

inv_p is functorial, well-defined, upper-semi-continuous.

- $a_1 = \text{ord}$, $\mathcal{D}(\mathcal{I})$ are functorial for smooth maps $\Rightarrow C(\mathcal{I}, a_1)$ is functorial,
- so by induction $\text{inv}_p(\mathcal{I}, x_1, \ldots, x_n)$ is functorial.
- Note that $\text{inv}_p(\mathcal{I}, x_1, \ldots, x_n) = \text{inv}_p(\mathcal{I}, x_1, x_2 + tx_1, \ldots, x_n + tx_1)$.

inv_p is functorial, well-defined, upper-semi-continuous.

- $a_1 = \text{ord}$, $\mathcal{D}(\mathcal{I})$ are functorial for smooth maps $\Rightarrow C(\mathcal{I}, a_1)$ is functorial,
- so by induction $\text{inv}_p(\mathcal{I}, x_1, \ldots, x_n)$ is functorial.
- Note that $\text{inv}_p(\mathcal{I}, x_1, \ldots, x_n) = \text{inv}_p(\mathcal{I}, x_1, x_2 + tx_1, \ldots, x_n + tx_1)$.
- Choosing appropriate t we have $(x_1', x_2 + tx_1, \ldots, x_n + tx_1)$ system of parameters.

inv_p is functorial, well-defined, upper-semi-continuous.

- $a_1 = \mathrm{ord}$, $\mathcal{D}(\mathcal{I})$ are functorial for smooth maps $\Rightarrow C(\mathcal{I}, a_1)$ is functorial,
- so by induction $\mathrm{inv}_p(\mathcal{I}, x_1, \ldots, x_n)$ is functorial.
- Note that $\mathrm{inv}_p(\mathcal{I}, x_1, \ldots, x_n) = \mathrm{inv}_p(\mathcal{I}, x_1, x_2 + tx_1, \ldots, x_n + tx_1)$.
- Choosing appropriate t we have $(x_1', x_2 + tx_1, \ldots, x_n + tx_1)$ system of parameters.
- MC-invariance gives $\pi^*\mathcal{I}[2] = \pi'^*\mathcal{I}[2]'$.

inv_p is functorial, well-defined, upper-semi-continuous.

- $a_1 = \text{ord}$, $\mathcal{D}(\mathcal{I})$ are functorial for smooth maps $\Rightarrow C(\mathcal{I}, a_1)$ is functorial,
- so by induction $\text{inv}_p(\mathcal{I}, x_1, \ldots, x_n)$ is functorial.
- Note that $\text{inv}_p(\mathcal{I}, x_1, \ldots, x_n) = \text{inv}_p(\mathcal{I}, x_1, x_2 + tx_1, \ldots, x_n + tx_1)$.
- Choosing appropriate t we have $(x_1', x_2 + tx_1, \ldots, x_n + tx_1)$ system of parameters.
- MC-invariance gives $\pi^*\mathcal{I}[2] = \pi'^*\mathcal{I}[2]'$.
- By induction and fuctoriality $a_2, \ldots, a_n$ well defined.

inv_p is functorial, well-defined, upper-semi-continuous.

- $a_1 = \text{ord}$, $\mathcal{D}(\mathcal{I})$ are functorial for smooth maps $\Rightarrow C(\mathcal{I}, a_1)$ is functorial,
- so by induction $\text{inv}_p(\mathcal{I}, x_1, \ldots, x_n)$ is functorial.
- Note that $\text{inv}_p(\mathcal{I}, x_1, \ldots, x_n) = \text{inv}_p(\mathcal{I}, x_1, x_2 + tx_1, \ldots, x_n + tx_1)$.
- Choosing appropriate t we have $(x_1', x_2 + tx_1, \ldots, x_n + tx_1)$ system of parameters.
- MC-invariance gives $\pi^*\mathcal{I}[2] = \pi'^*\mathcal{I}[2]'$.
- By induction and fuctoriality $a_2, \ldots, a_n$ well defined.
- $(a_2, \ldots, a_n)$ USC on $V(x_1)$, containing the maximal locus of a_1,
- so inv_p USC.