The invariant of the coefficient ideal drops

Theorem

$C(\mathcal{I}, a_1)\mathcal{O}_{Y'} \;=\; E^{\ell'} C'$ *with* $\mathrm{inv}_{p'} C' \;<\; \mathrm{inv}_p(C(\mathcal{I}, a_1))$.

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- Use formal decomposition $C(\mathcal{I}, a_1) = (x_1^{a_1!}) + (x_1^{a_1!-1})\tilde{\mathcal{G}}_1 + \cdots + (x_1)\tilde{\mathcal{G}}_{a_1!-1} + \tilde{\mathcal{G}}_{a_1!}$.
- If $H = V(x_1)$ then the proper transform $H' \to H$ is, up to rescaling, the blowing up of $J_H = (x_2^{a_2}, \ldots x_k^{a_k})$.
- In the x_1-chart the term $(x_1^{a_1!})$ becomes principal, so $\mathrm{inv}_{p'}(C') = 0$.
- In other charts the term $(x_1^{a_1!})$ transforms to $({x'_1}^{a_1!})$.
- So $\mathrm{ord}(C') \leq a_1!$, and we may assume equality, and x'_1 is maximal contact.
- Induction gives $\mathrm{inv}_{p'}(((\mathcal{G}_{a_1!})_H)') < (a_1 - 1)!(a_2, \ldots, a_k)$, so together the result follows.

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- One relies on inclusions [BM]: $\mathcal{I}'^{(a_1-1)!} \subset C' \subset C(\mathcal{I}', a_1)$.
- Hence $\mathrm{ord}\mathcal{I}' \leq a_1$, and we may assume equality with x_1' a maximal contact.
- Now $\mathrm{inv}_{p'}(\mathcal{I}'^{(a_1-1)!}) \geq \mathrm{inv}_{p'}(C') \geq \mathrm{inv}_{p'}(C(\mathcal{I}', a_1))$.
- By unique admissibility $\mathrm{inv}_{p'}(\mathcal{I}'^{(a_1-1)!}) = \mathrm{inv}_{p'}(C(\mathcal{I}', a_1))$ giving equalities throughout.
- By the previous theorem $\mathrm{inv}_{p'}\mathcal{I}' \ < \ \mathrm{inv}_p(\mathcal{I})$.