Brown University Vector Boot Camp - Practice Problems

1. Let $\vec{v} = \langle 5, 5\sqrt{3} \rangle$ and $\vec{w} = \langle 6, 2\sqrt{3} \rangle$. For parts (a) through (e), evaluate each of the following expressions.
   (a) $4\vec{v}$
   (b) $\vec{w} - \vec{v}$
   (c) $\vec{v} \cdot \vec{w}$
   (d) $\vec{w} \cdot \vec{v}$
   (e) $|\vec{w}|$
   (f) Determine the angle between $\vec{v}$ and $\vec{w}$.

2. Let $\vec{v} = \langle 3, 8, -1 \rangle$ and $\vec{w} = \langle 5, 1, 4 \rangle$. Evaluate each of the following expressions.
   (a) $\vec{v} + \vec{w}$
   (b) $3\vec{v} - 10\vec{w}$
   (c) $|\vec{v}|$
   (d) $\vec{v} \cdot \vec{w}$
   (e) $\vec{v} \times \vec{w}$
   (f) $\vec{w} \times \vec{v}$
   (g) $\vec{v} \times (3\vec{v} - 10\vec{w})$

3. Let $\vec{v} = \langle 1, 2, 3 \rangle$ and $\vec{w} = \langle -5, 4, 5 \rangle$.
   (a) Find the scalar projection of $\vec{v}$ onto $\vec{w}$.
   (b) Find the vector projection of $\vec{w}$ onto $\vec{v}$.
   (c) Write $\vec{v}$ as the sum of a vector parallel to $\vec{w}$ and a vector perpendicular to $\vec{w}$.

4. A hiker walks eight miles north, then walks two miles east, then walks three miles south, and then walks five miles west. A friend of the hiker, standing in the position where the hiker began, is trying to find the hiker using a set of binoculars.
   (a) How far is the hiker from his friend?
   (b) Suppose that the hiker texted his friend after each of his four walks, but the second of the four messages was not sent, so she believes he only moved north, south, and west. She incorrectly aims her binoculars at what she thinks is the hiker's position. How far will she have to rotate in order to find the hiker's actual position? Express your answer using an inverse trigonometric function.

5. A 50-pound weight is suspended from a flat ceiling by two cords. The angle between one cord and the ceiling is 60 degrees, and the angle between the other cord and the ceiling is 45 degrees. The force applied to the weight by each cord is called tension, and the system is in equilibrium, meaning that the net force applied to the weight (combining the force of gravity and the two tensions) is zero. Determine the magnitude of each of the tensions, in pounds.

(continued on next page)

6. Let $\vec{v} = \langle 1, -7, 0 \rangle$ and $\vec{w} = \langle 2, 6, -3 \rangle$.

   (a) Find a unit vector that points in the opposite direction of $\vec{w}$.

   (b) Find two unit vectors that are perpendicular to both $\vec{v}$ and $\vec{w}$.

7. Let $\vec{u} = \langle 1, 3, -2 \rangle$, $\vec{v} = \langle 7, -1, 2 \rangle$, and $\vec{w} = \langle -2, 8, 11 \rangle$.

   (a) Calculate the magnitude of each of these vectors.

   (b) Confirm that these three vectors are pairwise orthogonal (or in other words, any two of them are orthogonal to each other).

   (c) Consider the parallelepiped generated by these three vectors. Using your answers to parts (a) and (b), determine its volume.

   (d) Calculate the same volume as in part (c) by instead using the box product (triple scalar product).