ERRATA FOR "A SHARP CONDITION FOR SCATTERING OF THE RADIAL 3D CUBIC NONLINEAR SCHRÖDINGER EQUATION"

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Abstract. We correct a mistake with the Strichartz estimates in our CMP paper. We thank Cristi Guevara and Fernando Carreon for pointing out this mistake.

$(\frac{5}{3}, 10)$ is not an L^2 admissible Strichartz pair. Thus, our article needs straightforward modification in the following places.

0.1. **Modifications to the proof of Prop. 2.1.** *Starting with the line "Applying the Strichartz ..." replace by the following:* Applying the Strichartz estimates, we obtain

$$\|D^{1/2}\Phi_{u_0}(v)\|_{S(L^2)} \le c\|u_0\|_{\dot H^{1/2}} + c\|D^{1/2}(|v|^2 v)\|_{L_t^{10/7}L_x^{10/7}}$$

and

$$\|\Phi_{u_0}(v)\|_{S(\dot H^{1/2})} \le \|e^{it\Delta}u_0\|_{S(\dot H^{1/2})} + c\|D^{1/2}(|v|^2 v)\|_{L_t^{10/7}L_x^{10/7}}\,.$$

Applying the fractional Leibnitz [18] and Hölder inequalities,

$$\|D^{1/2}(|v|^2 v)\|_{L_t^{10/7}L_x^{10/7}} \le \|v\|^2_{L_t^5 L_x^5}\|D^{1/2}v\|_{L_t^{10/3}L_x^{10/3}} \le \|v\|^2_{S(\dot H^{1/2})}\|D^{1/2}v\|_{S(L^2)}\,.$$

The remainder of the proof is the same.

0.2. **Modifications to the proof of Prop. 2.2.** *At the beginning of the proof, insert the following lines:*

Let $M = \|u\|_{L_t^5 L_x^5} < \infty$. We claim that $\|\langle\nabla\rangle u\|_{S(L^2)} \lesssim BM^5$, where $\langle\nabla\rangle = (I-\Delta)^{1/2}$. Decompose $[0,+\infty) = \cup_{j=1}^{\tilde M} I_j$, where $\tilde M \sim M^5$, and for each j, $\|u\|_{L^5_{I_j}L^5_x} \le \delta$. Applying $\langle\nabla\rangle$ to the integral equation on I_j and then applying the Strichartz estimates, we obtain

$$\begin{aligned}\|\langle\nabla\rangle u\|_{S(L^2;I_j)} &\le B + \|u^2\langle\nabla\rangle u\|_{L^{10/7}_{I_j}L^{10/7}_x}\\ &\le B + \|u\|^2_{L^5_{I_j}L^5_x}\|\langle\nabla\rangle u\|_{L^{10/3}_{I_j}L^{10/3}_x}\\ &\le B + \delta^2\|\langle\nabla\rangle u\|_{S(L^2;I_j)}\end{aligned}$$

From this we conclude

$$\|\langle\nabla\rangle u\|_{S(L^2;I_j)} \le 2B$$

and by summing over the $\tilde M$ intervals, we conclude the proof of the claim.

After the definition of ϕ_+, replace the rest of the proof with the following: We first show that $\phi_+ \in H^1$. Indeed, by the Strichartz estimates and the definition of ϕ_+,

$$\begin{aligned}
\|\phi_+\|_{H^1} &\leq \|u_0\|_{H^1} + \|u^2 \langle \nabla \rangle u\|_{L^{10/7}_{[0,+\infty)} L^{10/7}_x} \\
&\leq \|u_0\|_{H^1} + \|u\|^2_{L^5_{[0,+\infty)} L^5_x} \|\langle \nabla \rangle u\|_{L^{10/3}_{[0,+\infty)}} L^{10/3}_x \\
&\lesssim B + BM^7
\end{aligned}$$

Applying the Strichartz estimates to (2.3),

$$\begin{aligned}
\|u(t) - e^{it\Delta}\phi_+\|_{H^1} &\leq c\|u^2 \langle \nabla \rangle u\|_{L^{10/7}_{[t,+\infty)} L^{10/7}_x} \\
&\leq \|u\|^2_{L^5_{[t,+\infty)} L^5_x} \|\langle \nabla \rangle u\|_{L^{10/3}_{[t,+\infty)} L^{10/3}_x}
\end{aligned}$$

and the right side $\to 0$ as $t \to +\infty$.

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