MATH 1530 ABSTRACT ALGEBRA
PROBLEM SET 11, DUE TUESDAY APRIL 25 1PM IN CLASS

1. Dummit and Foote pp. 277-278 problems 1(a), 1(b), 3

2. Let K be a field. Prove that $K[[x]]$ is a Euclidean domain with respect to the following norm: $N(0) = 0$, and for all nonzero $p \in K[[x]]$, $N(p)$ is the *order* of p, i.e. the smallest exponent appearing in p.

3. Dummit and Foote p. 283 problem 5

4. Compute a gcd of $4 + 2i$ and $5i$ in $\mathbb{Z}[i]$. Identifying $\mathbb{Z}[i]$ with the integer lattice points in the complex plane, draw a picture of the elements of the ideal $(4 + 2i, 5i)$.

5. Let R be an integral domain. We defined the *field of fractions* K, whose elements are equivalence classes of $\{(a, b) \colon a, b \in R, b \neq 0\}$ where $(a, b) \sim (c, d)$ if $ad = bc$. We write a/b for the class of (a, b). We defined

$$a/b + c/d = (ad + bc)/bd, \qquad a/b \cdot c/d = (ac)/(bd).$$

Convince yourself that $+$ and $\cdot$ are well-defined and make K into a field with $0 = 0/1$ and $1 = 1/1$ (ungraded).

   (a) Prove that the map $i \colon R \to K$ given by $i(r) = r/1$ is a ring homomorphism sending all nonzero elements to units.

   (b) Prove the following *universal property of localization*: Let S be a commutative ring with 1. If $f \colon R \to S$ is any ring homomorphism sending all nonzero elements of R to units of S, then there is a unique ring homomorphism $\widetilde{f} \colon K \to S$ such that

$$f = \widetilde{f} \circ i.$$