MATH 1530 ABSTRACT ALGEBRA
PROBLEM SET 4, DUE TUESDAY FEBRUARY 28 1PM IN CLASS

1. Dummit and Foote Problems 1 and 2 on page 48.

2. How many homomorphisms $\mathbb{Z} \to \mathbb{Z}/3\mathbb{Z}$ are there? How many homomorphisms $\mathbb{Z}/3\mathbb{Z} \to \mathbb{Z}$ are there? Justify your answers briefly.

3. Consider the additive group $\mathbb{Z}^2$, pictured as the lattice points in the Cartesian plane. Draw pictures of the following subgroups of $\mathbb{Z}^2$:

   (a) $\langle (2,1),(0,1)\rangle$
   (b) $\langle (1,1),(-1,1)\rangle$
   (c) $\langle (2,1),(1,1)\rangle$

4. Let $(a,b),(c,d) \in \mathbb{Z}^2$. Prove that $\langle (a,b),(c,d)\rangle = \mathbb{Z}^2$ if and only if

$$\det \begin{pmatrix} a & c \\ b & d \end{pmatrix} = \pm 1.$$

5. Does $\mathbb{R}$ have any subgroups isomorphic to $\mathbb{Z}^2$? Prove your answer.

6. Does $\mathbb{Q}$ have any subgroups isomorphic to $\mathbb{Z}^2$? Prove your answer.

7. Dummit and Foote problems 2 and 11 on page 60.