MATH 1530 ABSTRACT ALGEBRA
PROBLEM SET 5, DUE TUESDAY MARCH 7 1PM IN CLASS

1. Dummit and Foote problem 16 and 18 on page 45

2. (a) Let G be a group acting on a set A. The *stabilizer* of an element $a \in A$, denoted G_a, is defined to be the set
$$G_a = \{g \in G : g \cdot a = a\}.$$
Prove that the stabilizer is a subgroup of G.

   In class on Thursday February 23, you considered the action of the group G of rotations on the power set $\mathcal{P}(\mathbb{R}^2)$ of $\mathbb{R}^2$.

   (b) Which elements of $\mathcal{P}(\mathbb{R}^2)$ have stabilizer equal to G? Justify your answer briefly.

   (c) (optional, extra) Does there exist any $S \in \mathcal{P}(\mathbb{R}^2)$ with infinite cyclic stabilizer?

3. Dummit and Foote problem 9 on page 71.

4. Let p be any prime number. Prove that every group of order p is isomorphic to $\mathbb{Z}/p\mathbb{Z}$.

5. We showed in an earlier lecture that
$$Z(G) = \{x \in G \ : \ gx = xg \text{ for all } g \in G\}$$
is a subgroup of G, called the *center* of G. Prove that $Z(G)$ is normal, and that it consists precisely of the elements of G whose conjugacy classes have size 1.

6. List the conjugacy classes of the dihedral group D_{12}. Draw the lattice of subgroups of D_{12} and indicate in your lattice which subgroups are normal. (No proofs necessary.)