MATH 1530 ABSTRACT ALGEBRA
PROBLEM SET 9, DUE TUESDAY APRIL 11 1PM IN CLASS

1. Dummit and Foote page 230–232, problems 5, 14
2. Dummit and Foote page 238–239, problems 3, 10
3. Dummit and Foote page 248, problem 10. Also, in parts (a) and (e), identify the quotient of $\mathbb{Z}[x]$ by the given ideal as isomorphic to a more familiar ring (no proofs necessary for this).
4. Dummit and Foote page 250 problem 29. Also, let I be the ideal of $\mathbb{Z}[x]$ consisting of polynomials whose constant term, coefficient of x, and coefficient of x^2 are zero. Identify, with proof, the nilradical of $\mathbb{Z}[x]/I$.
5. Let I be an ideal in a ring R, and let $\pi\colon R \to R/I$ be the natural projection. Prove the following *universal property of quotients:* If $\varphi\colon R \to S$ is any ring homomorphism such that $I \subseteq \ker(\phi)$, then there exists a unique homomorphism $\overline{\varphi}\colon R/I \to S$ such that

$$\varphi = \overline{\varphi} \circ \pi.$$