Geometry and Illustration: Syllabus
Tu-Th 1:00-2:20
ICERM Main Lecture Hall
Richard Schwartz (res@math.brown.edu)
Office Hours: TBD

Course Description: This class is a survey of topics in geometry and dynamics, with an emphasis on topics that are amenable to computer experimentation and visual programming. (And, honestly, there is an emphasis on topics which I already somewhat know and can exposit fairly easily.) Along with each topic, I will give students some ideas for making programs to further investigate the topic. The class is meant to be fairly modular, in that I will try to treat each topic more or less from scratch. Below is a week-by-week list of topics.

Caveat: The topics list below is probably somewhat more material than I can cover, especially because the course may not run during the 3 weeks when we have outside workshops. Also, I'm going to be at a conference in Oxford for the week of Sept 23-28, so Bill Goldman will give substitute lectures on a topic of his choice. (I'm sure it will be something in geometry/dynamics.)

Grading and HW: There is no HW *per se*, though I hope that you get involved in at least one project and love it! For Brown students: you will pass the class provided that you attend semi-regularly, engage with the class somewhat, and still have a heartbeat at the time I have to turn in the grades. For other ICERM participants: there are no grades.

Week 1: Euclidean Geometry

- Lecture 1: Euclidean space, isometries, 3D platonic solids, quaternions, the spin cover, 4-dimensional platonic solids.
- project ideas: Visualize the spin cover. Render Euclidean polyhedra. Check the properties of the $E8$ cell computationally. Combinatorics of spherical designs, Steiner systems.

Week 2: Euclidean Surfaces and Tilings

- Lecture 1: The Voronoi decomposition, flat tori, planar Euclidean orbifolds, translation surfaces.
- Lecture 2: The Penrose Tilings: inflation, aperiodicity, pentagrids, cut-and-project.
- Project ideas: Make a symmetry analyzer. Render the Penrose tilings. Investigate the Amman-Beenker tilings, the Robinson tilings, the Taylor-Socolar tiling, the pinwheel tiling.

Week 3: Hyperbolic Geometry

- Lecture 1: Lecture 1: Basics: Poincare, Klein, and Lorenz models.
- Lecture 2: Lecture 2: Special groups: modular group, pythagorean triples, the Appollonian gasket.
- Project ideas: Computing fundamental domains for congruence subgroups. Rendering the Appollonian gasket. Investigating continued fractions and the Gauss map. Making a hyperbolic viewer.

Week 4: Hyperbolic Structures.

- Lecture 1: Surface groups and some Hyperbolic 3-manifold examples. Dehn Surgery.
- Lecture 2: Intro to Teichmuller space.
- Project ideas: Drawing pants decompositions. Mapping class action computer. Using Snappea. Experiments with trace coordinates. Studying arithmetic examples computationally.

Week 5: Exotic Geometric Structures I.

- Lecture 1: Projective Geometry.
- Lecture 2: Convex projective structures.
- Project ideas: Drawing in perspective. Drawing domains for convex projective structures. Investigating the Hitchen component computationally. Rendering classical theorems in projecrtive geometry.

Week 6: Exotic Geometric Structures II:

- Lecture 1: Lorenz spacetimes and crooked planes.
- Lecture 2: Complex hyperbolic geometry and triangle groups.
- Project ideas: rendering crooked planes, drawing in complex hyperbolic geometry, drawing limit sets. Experiments with traces and trace coordinates.

Week 7: Riemann mapping and uniformization.

- Lecture 1: Basic definitions, rapid primer on complex analysis.
- Lecture 2: Circle packing proof of the Riemann mapping Theorem.
- Project ideas: Thurston's algorithm for circle packings. Stevenson's algorithm. The zipper method for the Riemann map. Finite element methods. Circle uniformization of slit domains. Programming the Schwarz-Christoffel transform.

Week 8: Complex dynamics

- Lecture 1: Basic definitions: Julia set, Newton's method, Mandelbrot set.
- Lecture 2: Further properties of the Mandelbrot set: Connectness, Riemann map, external rays.
- Project ideas: Study the dynamics of Newton's method. Study the Doyle-McMullen root-finding algorithm. Render the Mandelbrot set yourself. Draw the Mandelbox set.

Week 9: Euclidean Dynamics I

- Lecture 1: Interval Exchange Transformations and Rauzy Renormalization.
- Lecture 2: Rational Billiards, translation surfaces (again) and the Veech Group.
- Project ideas: Implement Rauzy Renormalization. Make a billiards program. Discussion of the Hooper-Delaxroix Veech group computer. Billiards in irrational polygons. Polyhedral billiards. Affine and Lorenzian billiards.

Week 9: Euclidean Dynamics II

- Lecture 1: Outer Billiards basics: tilings, periodic islands, existence of periodic orbits.
- Lecture 2: My work on outer billiards on kites.
- Project ideas: Make an outer billiards interface. Hyperbolic outer billiards. Symplectic outer billiards on the 24-cell.

Week 10: P-adic numbers and group actions

- Lecture 1: The basics: definition of p-adic numbers, Hensel's Lemma, odometer, solenoid.
- Lecture 2: The Serre Tree and group actions on products of Serre trees.
- Project ideas: Make a p-adic visual calculator. Compute the action on the Serre trees. Implement Hensel's Lemma. Visualize tree automorphisms generally.

Week 11: Energy minimization and flows

- Lecture 1: Curve Shortening, mean curvature flow.
- Lecture 2: Energy minimization, gradient flow, hill climbing.
- Projecti ideas: Make a curve-shortening program. Make an energy minimization program.