Math 1040 HW2:

1: Prove that it is not possible to dissect the square into 3 convex polygons and then rearrange them so that they form an equilateral triangle.

2: Call a (solid) polygon P of area 1 *tame* if it has the following properties:

1. P is convex.
2. P has at most 100 sides.
3. P contains a disk of radius $1/100$.

Prove that there is some universal number N such that any two tame polygons can be dissected into each other using less than N pieces.

3: Show that the conclusion of Problem 2 is false if any one of the 3 conditions is dropped and you insist that the pieces are convex.

4: An *orthoscheme* is a tetrahedron whose vertices have the form $(0,0,0)$, and $(a,0,0)$ and $(a,b,0)$ and (a,b,c) for constants $a,b,c>0$. An *orthoscheme partition* of a polyhedron is a way of writing the polyhedron as a finite union of orthoschemes which have disjoint interiors. Prove that the cube and the regular tetrahedron both have orthoscheme partitions.

5: Prove that the octahedron has nonzero Dehn invariant. That means you can't dissect a cube into any platonic solid except itself. (Hint: this boils down to showing that the dihedral angle is irrational. Try to compare the dihedral angle for the regular octagon with the dihedral angle for the regular tetrahedron.) For extra credit, find an expression for the dihedral angles of the regular dodecahedron and the regular icosahedron.