Here are some HW problems about the Jordan Curve Theorem and related matters. For simplicity, I'll just give you the polygonal versions of the problems. Say that a *planar graph* is a 1-dimensional simplicial complex in the plane.

1. A *tree* is a planar graph with no circuits. Prove that the complement of a tree is connected. Figure 1 shows an example of a tree.

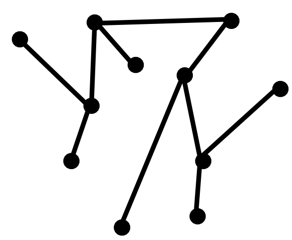

Figure 1: A tree

2. Let Θ be a planar graph that is homeomorphic to a graph with 2 vertices and 3 edges connecting them. Note that Θ might have many edges and vertices. Figure 2 shows an example of how Θ might look. Prove that $\boldsymbol{R}^2 - \Theta$ has exactly 3 connected components.

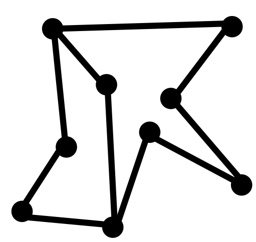

Figure 2: A possibility for Θ.

3. The graph $K_{3,3}$ has 3 white vertices and 3 black vertices, and an edge joining each white vertex to each black vertex. Prove that there is no planar graph homeomorphic to $K_{3,3}$. In other words, if you try to draw $K_{3,3}$ in the plane, then some edges will cross.

4. Prove that you can draw $K_{4,4}$ in the torus without any edges crossing.