MATH 1720: Manifold
Tu-Th 2:30-3:50)
Barus & Holley 751
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U.T.A.: Nathan Smith
Office Hours: TBD

Course Description: This is a first course in manifolds. It covers many examples of manifolds, such as spheres, tori, higher genus surfaces, 3-manifolds, and projective spaces. The course also develops the theory of the basic objects associated to manifolds: vector and tensor fields, differential forms, and Riemannian metrics. One highlight of the course is the proof of Stokes' Theorem on manifolds. Advanced topics may include DeRham cohomology, Lie groups, and hyperbolic manifolds.

Objectives: Here are 4 main objectives for the class.

1. Learn the course material.
2. Solve challenging problems related to the course material.
3. Practice writing mathematical proofs.
4. Develop an intuition for manifolds and space in general.

Grading: Your grade has 3 components.

- HW: 20%
- Midterm 30%
- Final: 50%

We will discuss the situation one-on-one in case you miss one of the exams and have a legitimate excuse.

HW Assignments: There will be weekly HW assignments. The assignments will consist of about 5 problems. I will send you email each week about the assignments, and also post them on the course website: (http://www.math.brown.edu/∼res/M1720) each Tuesday and then collect them the next Tuesday in class. There is no late HW allowed.

You are allowed to discuss the HW with other people in the class, but I think that it is better for you to do the HW largely on your own. If you do discuss the HW with other people, you should list their names on your assignment when you turn it in. Given that this is 2026, chatGPT and other AIs probably do most of your HW pretty well. You should remember that the object of the HW is not for you to complete the assignment but rather for you to strengthen your understanding of the material. If you use AI for any purpose on your HW you need to detail the help you get in the write-up of the HW.

Exams: We will decide the dates of the midterms by class vote. I hope to have the midtern around the 7th week. I will hold the midterm in the evening and it will last 3 hour. The purpose of having a long-format in-class exam is remove time pressure from the exam without having the uncontrolled environment associated with a take-home exam. The final exam will be an in-class exam lasting 3 hours.

Readings: The book for the course is *An Introduction to Manifolds* by Loring Tu. Mostly this book will be a reference. I will not follow the book in order, but will try to explain where in the book (when applicable) you can find the material from the lecture. I will also give out notes I have written.

Accommodations: Brown University is committed to full inclusion of all students. Please inform me, as soon as possible, if you have any conditions which might require special consideration. For more information, please contact the Student and Employee Accessibility Services (SEAS) at SEAS@brown.edu or 401.863.9688.

Zoom Days: There will be a 3 days when I am traveling, and on those days I'm planning to give Zoom lectures. Those days are:

- March 10
- April 21
- April 23

I hope this will work out OK. If the first two are a terrible experience, we can skip the last one and make it up during reading period.

Course content by Lecture: This is tentative list of topics. The pacing of the course may depend on questions that arise in class, holidays, and other things.

Part 1: Definition of a Manifold and many examples: (8 lectures)

1. Metric and topological spaces, definition of a topological manifold
2. Diffeomorphisms, definition of a smooth manifold.
3. Surfaces: sphere, torus, projective plane, connect sum operation.
4. Higher dimensional spheres, tori, projective spaces, Grassmannians
5. The 3-sphere in detail: $SO(3)$, quaternions, spin cover.
6. The Poincare manifold; four dimensional platonic solids
7. 3D Constructions: mapping tori, Dehn filling, Heegard splittings
8. The complex projective plane in detail: Trisection description

Part 2: Embedded manifolds (4 Lectures)

1. Prep for the Inverse Function Theorem: Sperner's Lemma, retractions
2. Proof of the Inverse and Implicit Function Theorem
3. Using the Implicit Function Theorem to construct manifolds.
4. Examples: algebraic surfaces, Veronese embedding, Boy's surface

Part 3: Objects Associated to Manifolds (8 Lectures)

1. Tangent vectors and the tangent space of a manifold.
2. Vector fields on manifolds. Hairy Ball Theorem
3. Tensors, tensor product, and tensor fields on manifolds
4. Riemannian manifolds: hyperbolic space, Fubini-Study metric

5. Alternating Tensors and the wedge product
6. Differential forms on manifolds and the d-operator.
7. Partitions of Unity
8. Integration of Differential Forms

Part 4: Stokes' Theorem (4 Lectures)

1. Transformation Law for the d operator.
2. Change of Variables formula for integration of forms.
3. Manifolds with boundary; orientation consistency
4. Proof of Stokes Theorem.

Part 5: Advanced Topics. This will depend on how much time we have left and on the interests of the class. Possible topics:

- De Rham Cohomology
- The Hodge Star operator and harmonic forms.
- Lie groups
- hyperbolic manifolds
- Nash embeddings