Math 20 Midterm 2. 19 Nov 2010
Instructions. The problems are worth 25 points each. Show all your work.

1. Evaluate the integral

$$\int_{x=0}^{1} \int_{y=\sqrt{x}}^{1} x \cos(\pi y^5/2)\ dy\ dx.$$

You will find it useful to sketch the domain of integration and also switch the order of integration.

2. Write down a double integral, in polar coordinates, that computes the surface area of the portion of the paraboloid $z = x^2 + y^2$ that lies underneath the plane $z = 2x$. You don't have to evaluate the integral. The point of this problem is to find the correct integrand and set up the limits of integration.

3. a. (10 pts) Consider the half-disk whose diameter joins the points $(0, 0)$ and $(2, 0)$ and which lies above the x-axis. See the left side of Figure 1. Compute the centroid of the half-disk by combining Pappus's theorem with the fact that the ball of radius 2 has volume $4\pi/3$.

b. (15 pts) Let R be the region obtained by attaching 3 half-disks to the sides of a square, as in Figure 1. The sides of the square are 2 meters. All objects have uniform density. Compute the distance from the centroid of R to the center of the square.

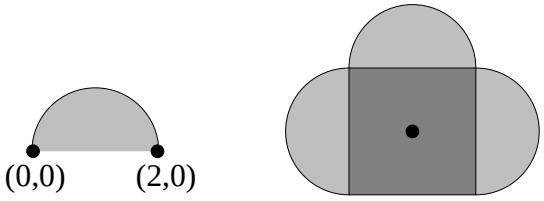

Figure 1: The region R.

4. Let R be the planar region in the (u, v) plane that is bounded by the hyperbolas $uv = 1$ and $uv = 2$ and $u^2 - v^2 = 4$ and $u^2 - v^2 = 8$. Use the change of variables formula $(x, y) = (u^2 - v^2, 2uv)$ to compute the integral

$$\int_R u^2 + v^2\ du\ dv.$$

Note: You don't need to compute the "inverse Jacobian" in this problem.