## Math 20 Midterm 3. 7 Apr 2008

Instructions. Each numbered problem is worth 25 pts. Show all your work.

1. Evaluate the following integral:

$$\int \int_R 2y \, dy \, dx$$

where R is the half-disk given by  $x^2 + y^2 \leq 1$  and  $y \geq 0$ .

2. Figure 1 shows a region made from a square and a right triangle. Assume that the region has uniform density. Compute the distance from the center of mass (i.e. centroid) of the region to the bottom left corner. Most of the sides have length 1, as indicated in the figure.



Figure 1

**3.** Let R be the region in space that lies below  $z = 18 - x^2 - y^2$  and above  $z = x^2 + y^2$ . (a) Set up the integral

$$\int \int \int_R x^2 \ dV$$

in cylindrical coordinates. This means that you should find the limits of integration and also the function to be integrated. You don't need to evaluate the integral.

(b) Compute the surface area of  $\partial R$ .

**4.** Consider the coordinate change F(u, v, w) = (2u, u + v, u + v + 3w). Let S be the unit cube, bounded by u = 0, u = 1, v = 0, v = 1, w = 0, w = 1. Let R = F(S). Compute the integral

$$\int \int \int_R y^2 \ dV$$

by applying the change of variables formula and integrating the appropriate function over S.