Math 2410 HW2

Due Tuesday, Nov 2.

1: This is closely related to Problem 1.2.4 in Hatcher's book: Let X_k be the union of a sphere in $\boldsymbol{R}^3$ and k disjoint chords connecting points on the boundary of the sphere. Construct universal covers for X_1 and X_2.

2: This is problem 1.2.13 in Hatcher's book: Let X be the figure 8. Determine the covering space of X curresponding to the subgroup of $\pi_1(X)$ generated by the cubes of all the elements. (So, a^3 is in the subgroup and $(ab)^3$ is in the subgroup, and so on.)

3: This is problem 23 in Hatcher's book: A group G acts *freely* on a space X if no element of G fixes and points of X. The group G acts *pproperly discontinuously* if each point $x \in X$ an open neighborhood U such that all but finitely many elements of G move U completely off itself. That is $g(U) \cap U \neq \emptyset$ for only finitely many g. Prove that if X is Hausdorff and G acts freely and properly discontinuously on X then the quotient map $X \to X/G$ is a covering space.

4: Put a Δ-complex structure on the Klein bottle and compute its simplicial homology. (This is basically Problem 2.1.5 in Hatcher's book.)

5: Prove that $\boldsymbol{CP}^2$ is homeomorphic to a simplicial complex. (Hint: One idea would be to try to use the cell decomposition as a starting point and then improve it until it is a simplicial complex.) In fact, any compact smooth manifold is homeomorphic to a simplicial complex, so one approach to this problem would be to just do it all at once for any smooth manifold.

6: Show that a lens space is homeomorphic to a simplicial complex and then compute its simplicial homology. (See Problem 2.1.8 in Hatcher's book.)