Math 2410 HW3

Due Thursday, Dec 2. Do 6 out of the first 7 questions.

1: Compute the homology groups of an oriented surface of genus g for each $g = 1, 2, 3, \ldots.$

2: Compute the homology groups of $\boldsymbol{CP}^n$ for all $n \geq 1$ by using the long exact sequence for quotients. If you use cellular homology it is much easier.

3: Show that if A is a retract of X then the map $H_n(A) \to H_n(X)$ includes by the inclusion $A \subset X$ is injective. This is problem 2.1.11 in Hatcher's book.

4: Compute the relative homology groups $H_n(X, A)$ where X is the 2-sphere and A is a finite union of points in X. (This is part of Problem 2.1.17 in Hatcher's book.)

5: Given a map $f : S^{2n} \to S^{2n}$ show that there is a point $x \in S^{2n}$ such that either $f(x) = x$ or $f(x) = -x$. Deduce that every map from $\boldsymbol{RP}^{2n}$ to $\boldsymbol{RP}^{2n}$ has a fixed point. This is part of Problem 2.2.2 in Hatcher's book.

6: A polynomial $f(z)$ with complex coefficients defines a map of the Riemann sphere $\boldsymbol{C} \cup \infty$. Prove that the degree of this map equals the degree of this polynomial. This is part of Problem 2.2.8 in Hatcher's book.

7: Let M be a smooth compact manifold. Prove that there is a continuous surjective map from M to M which is homotopic to the constant map. This is a generalization of the problem in Hatcher's book about spheres and degree 0 maps.

Bonus Question: What is the height of the shortest giant who could walk across Rhode Island from Westerly to Woonsooket only stepping on the rooftops of Dunkin Donuts?