Math 2410 HW3

Due Thursday, Dec 8. Do 6 out of 7 questions.

1: Compute the homology groups of an oriented surface of genus g for each $g = 1, 2, 3, ...$

2: Compute the homology groups of $\boldsymbol{CP}^n$ using cellular homology and then using the long exact sequence for quotients.

3: Compute the relative homology groups $H_n(X, A)$ where X is the 2-sphere and A is a finite union of points in X. (This is part of Problem 2.1.17 in Hatcher's book.)

4: Prove that there are infinitely many homeomorphisms of the torus which are not homotopic to each other.

5: Prove that S^{2n} does not have a nowhere vanishing vector field. Use this result to prove the following statement. Given a map $f : S^{2n} \to S^{2n}$, either is a point x such that $f(x) = x$ or there is a point x such that $f(x) = -x$. (The second statement is part of Problem 2.2.2 in Hatcher's book.)

6: Let X be a topological space and let ΣX denote the suspension of X. This space is the join of X with the space is that is 2 distinct points. Prove that $H_{n+1}(\Sigma X) \cong X_n(X)$. Use this fact to show that for any finite list $G_1, ..., G_n$ of finite abelian groups there is a space X such that $H_k(X) \cong G_k$ for $k = 1, ..., n$.

7: A polynomial $f(z)$ with complex coefficients defines a map of the Riemann sphere $\boldsymbol{C} \cup \infty$. Prove that the degree of this map equals the degree of this polynomial. (This is part of Problem 2.2.8 in Hatcher's book.)