

## ERRATA TO “LINEAR ALGEBRA DONE WRONG”

- p. 30:** the definition of a subspace should read “A *subspace* of a vector space  $V$  is a non-empty subset  $V_0 \subset V$  of  $V$ ...”—the word non-empty is inserted.
- p. 31, problem 7.2:** the last sentence of the problem should read: “Show that if  $X$  and  $Y$  are *subspaces* of  $V$ , then  $X + Y$  is also a subspace.”
- p. 42:** line 3 from below should read: “Make sure, by applying row operations of type 1 (row exchange), if necessary...”—not “type 2”.
- p. 48:** There should be “row” instead of “column” in line 9 from below. This line should read: “. . . if and only if there is a pivot in every row in echelon form of the matrix.”
- p. 50, line 2:** should be “columns” instead of “rows” here. This line should read: “. . . cannot exceed the number of columns,  $n \leq m$ .”
- p. 55:** Subsection 5.1 should start before Proposition 5.4. The phrase “The following statement will play an important role later.” should be the first sentence of this subsection.
- p. 66, Sect. 7.4:** in the second to last paragraph before the Remark,  $A$  and  $A_e$  are mixed up, and the indices in  $\mathbf{v}_r, \dots, \mathbf{v}_n$  are wrong (it should be  $\mathbf{v}_{r+1}, \dots, \mathbf{v}_n$ ). The correct paragraph should read as:  
 “To see that, let vectors  $\mathbf{v}_{r+1}, \dots, \mathbf{v}_n$  complete the rows of  $A_e$  to a basis in  $\mathbb{R}^n$ . Then, if we add to a matrix  $A_e$  rows  $\mathbf{v}_{r+1}^T, \dots, \mathbf{v}_n^T$ , we get an invertible matrix. Let call this matrix  $\tilde{A}_e$ , and let  $\tilde{A}$  be the matrix obtained from  $A$  by adding rows  $\mathbf{v}_{r+1}^T, \dots, \mathbf{v}_n^T$ . The matrix  $\tilde{A}_e$  can be obtained from  $\tilde{A}$  by row operations, so

$$\tilde{A}_e = E\tilde{A},$$

where  $E$  is the product of the corresponding elementary matrices. Then  $\tilde{A} = E^{-1}$  and  $\tilde{A}$  is invertible as a product of invertible matrices.”

- p. 68, problem 7.13:** There should be question mark “?” (without quotes), not “/” at the end of the last sentence.
- p. 86:** there should be  $\mathbf{v}_n$  instead of  $\mathbf{e}_n$  on the last line.
- p. 86, Problem 3.10:** The last matrix there should be

$$\begin{pmatrix} A & \mathbf{0} \\ * & I \end{pmatrix}$$

- p. 87, line 9:** the line should read  
 “. . .  $D(\mathbf{e}_{j_1}, \mathbf{e}_{j_2}, \dots, \mathbf{e}_{j_n})$  is zero, because there are two equal columns here.” (“columns” instead of “rows”)

**p. 88:** equation (4.2) should read

$$(4.2) \quad \det A = \sum_{\sigma \in \text{Perm}(n)} a_{\sigma(1),1} a_{\sigma(2),2} \cdots a_{\sigma(n),n} \text{sign}(\sigma),$$

(should be no commas between  $a_{\sigma(k),k}$ )

**p. 95:** In item c) of Problem 5.6 the formula should be " $A_n \cdot (x - c_0)(x - c_1) \cdots (x - c_{n-1})$ ", not " $A_n \cdot (x - c_0)(x - c_1) \cdots (x - c_n)$ "

**p. 126, Problem 2.3:** In item a) the summation should be  $\sum_{k=1}^n \cdots$ , not  $\sum_{k=1}^{\infty} \cdots$

**p. 129:** The last line in the proof of Proposition 3.3 should read

$$= (\mathbf{v}, \mathbf{v}_k) - \alpha_k (\mathbf{v}_k, \mathbf{v}_k) = (\mathbf{v}, \mathbf{v}_k) - \frac{(\mathbf{v}, \mathbf{v}_k)}{\|\mathbf{v}_k\|^2} \|\mathbf{v}_k\|^2 = 0.$$

not

$$= (\mathbf{v}, \mathbf{v}_k) - \alpha_k (\mathbf{v}_k, \mathbf{v}_k) = \frac{(\mathbf{v}, \mathbf{v}_k)}{\|\mathbf{v}_k\|^2} \|\mathbf{v}_k\|^2 = 0.$$

**p. 142–143:** Proof of Lemma 6.2 should read:

*Proof.* If  $U^*U = I$ , then by the definition of adjoint operator

$$(\mathbf{x}, \mathbf{x}) = (U^*U\mathbf{x}, \mathbf{x}) = (U\mathbf{x}, U\mathbf{x}) \quad \forall \mathbf{x} \in X.$$

Therefore  $\|\mathbf{x}\| = \|U\mathbf{x}\|$ , and so  $U$  is an isometry.

On the other hand, if  $U$  is an isometry, then by the definition of adjoint operator and by Theorem 6.1 we have for all  $\mathbf{x} \in X$

$$(U^*U\mathbf{x}, \mathbf{y}) = (U\mathbf{x}, U\mathbf{y}) = (\mathbf{x}, \mathbf{y}) \quad \forall \mathbf{y} \in X,$$

and therefore by Corollary 1.5  $U^*U\mathbf{x} = \mathbf{x}$ . Since it is true for all  $\mathbf{x} \in X$ , we have  $U^*U = I$ .  $\square$

**p. 145:** The second line of the proof of the proposition 6.5 should read " $UB\mathbf{x} = U(\lambda\mathbf{x}) = \lambda U\mathbf{x}$ , i.e.  $U\mathbf{x}$  is an eigenvector of  $A$ ." (" $UB\mathbf{x}$  should be instead of  $UA\mathbf{x}$ )

**p. 157:** Conclusion of Theorem 1.1 should read

"In other words, any  $n \times n$  matrix  $A$  can be represented as  $A = UTU^*$ , where  $U$  is a unitary, and  $T$  is an upper triangular matrix." (" $A = UTU^*$ ", not " $T = UTU^*$ ")

**p. 164, line 2:** Text in problem 2.6 should read "... has positive eigenvalues...", not "... has positive eigenvectors..."

**p. 164, Problem 2.14:** One "eigenvalues" should be deleted.

**p. 171, problem 3.6:** The part b) of the problem should read

b) " $\min_{\|\mathbf{x}\|=1} \|A\mathbf{x}\|$  and the vectors where the minimum is attained;"

(i.e. the condition should be  $\|\mathbf{x}\| = 1$ , not  $\|\mathbf{x}\| \leq 1$ ).

**p. 173, line 12:** it should be " $\mathbf{x} = (x_1, x_2, \dots, x_n)^T$ ", not " $\mathbf{v} = (x_1, x_2, \dots, x_n)^T$ " there.

**p. 174, formula (4.1):** The formula and the text around it should read:

... Since for  $\mathbf{x} = (x_1, x_2, \dots, x_n)^T$

$$(1) \quad A\mathbf{x} = \sum_{k=1}^r s_k x_k \mathbf{e}_k,$$

we can conclude that

$$\|A\mathbf{x}\|^2 = \sum_{k=1}^r s_k^2 |x_k|^2 \leq s_1^2 \sum_{k=1}^r |x_k|^2 = s_1^2 \cdot \|\mathbf{x}\|^2,$$

so  $\|A\mathbf{x}\| \leq s_1 \|\mathbf{x}\|$ .

**p. 175:** The last displayed formula on this page should read

$$\|A\|_2^2 = \text{trace}(A^*A) = \sum_{k=1}^r s_k^2.$$

(there should be  $\|A\|_2^2$ , not  $\|A\|_2$ ).

**p. 179:** formula on line 10 from below should read

$$\mathbf{x} := \text{Re } \mathbf{u} = (\mathbf{u} + \bar{\mathbf{u}})/2, \quad \mathbf{y} = \text{Im } \mathbf{u} = (\mathbf{u} - \bar{\mathbf{u}})/(2i),$$

not

$$\mathbf{x}_k := \text{Re } \mathbf{u} = (\mathbf{u} + \bar{\mathbf{u}})/2, \quad \mathbf{y} = \text{Im } \mathbf{u} = (\mathbf{u} - \bar{\mathbf{u}})/(2i),$$

**p. 183, Lemma 5.6:** in the second line of the statement of the lemma it should be written "... rotations  $R_1, R_2, \dots, R_N, N \leq n(n-1)/2 \dots$ ", not "... rotations  $R_1, R_2, \dots, R_n, n \leq n(n-1)/2 \dots$ ".