AN OPERATOR CORONA THEOREM.

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Abstract. In this paper some new positive results in the Operator Corona Problem are obtained in rather general situation. The main result is that under some additional assumptions about a bounded analytic operator-valued function F in the unit disc $\mathbb{D}$ the condition

$$F^*(z)F(z) \geq \delta^2 I \qquad \forall z \in \mathbb{D} \qquad (\delta > 0)$$

implies that F has a bounded analytic left inverse. Typical additional assumptions are (any of the following):

1. *The trace norms of defects $I - F^*(z)F(z)$ are uniformly (in $z \in \mathbb{D}$) bounded. The identity operator I can be replaced by an arbitrary bounded operator here, and F^*F can be changed to FF^*;*
2. *The function F can be represented as $F = F_0 + F_1$, where F_0 is a bounded analytic operator-valued function with a bounded analytic left inverse, and the Hilbert–Schmidt norms of operators $F_1(z)$ are uniformly (in $z \in \mathbb{D}$) bounded.*

It is now well-known that without any additional assumption, the condition $F^*F \geq \delta^2 I$ is not sufficient for the existence of a bounded analytic left inverse.

Another important result of the paper is the so-called *Tolokonnikov's Lemma* which says that a bounded analytic operator-valued function has an analytic left inverse if and only if it can be represented as a "part" of an invertible bounded analytic function. This result was known for operator-valued function such that the operators $F(z)$ act from a finite-dimensional space, but the general case is new.