Extrapolation on variable L^p spaces.

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The variable L^p spaces are a generalization of the classical Lebesgue spaces. Given an exponent function $p(\cdot) : \mathbb{R}^n \to [1, \infty)$, define $L^{p(\cdot)}$ to be the space of functions such that for some $\lambda > 0$,

$$\int_{\mathbb{R}^n} |f(x)/\lambda|^{p(x)}\, dx < \infty.$$

This is a Banach space when equipped with the norm

$$\|f\|_{p(\cdot)} = \inf \left\{ \lambda > 0 : \int_{\mathbb{R}^n} |f(x)/\lambda|^{p(x)}\, dx \leq 1 \right\}.$$

When $p(\cdot) = p_0$ then $L^{p(\cdot)}$ equals L^{p_0}. These spaces are interesting in their own right, and have applications to physics, PDEs and the calculus of variations.

I will discuss sufficient conditions on $p(\cdot)$ for the Hardy-Littlewood maximal operator to be bounded on $L^{p(\cdot)}$, and then show how this yields an extrapolation theorem in the scale of variable L^p spaces. As a consequence we get that a large variety of classical operators from harmonic analysis—singular integrals, fractional integrals, multipliers—are bounded on $L^{p(\cdot)}$ whenever the maximal operator is.