Homework assignment, Feb. 14, 2007.

Due Friday, 2/16 (collected)

1. I'll be collecting the problem # 3 on p. 55 from the previous assignment.

2. Prove that if a sequence $\{a_n\}_{n=1}^{\infty}$ is bounded, and $\lim\limits_{n\to\infty} b_n = 0$ then $\lim\limits_{n\to\infty} a_n b_n = 0$.

3. Prove that if $\lim\limits_{n\to\infty} a_n = a \neq 0$, then $\lim\limits_{n\to\infty} \dfrac{1}{a_n} = 1/a$.

   One of the essential steps here can be the following statement (which you will need to prove if you use it): if $\lim\limits_{n\to\infty} a_n = a \neq 0$, then the "tail" of the sequence $1/a_n$ is bounded, i.e. there exist numbers N and R such that for all $n > N$ the expression $1/a_n$ is defined (i.e. $a_n \neq 0$) and $|a_n| \leq R$.

4. State what does it mean that $\{a_n\}_{n=1}^{\infty}$ is not a Cauchy sequence.

   You cannot start your statement with negation, i.e. you cannot begin "There is no …". Your statement should start with one of the quantifiers, $\forall$ or $\exists$.