

Homework assignment, Feb. 23, 2007.

1. Let $D : \mathbb{R} \rightarrow \mathbb{R}$ be the Dirichlet function

$$D(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$$

Prove that D is not discontinuous (i.e. not continuous) at any point $x \in \mathbb{R}$.

2. Construct an example of a continuous functions $f, g : \mathbb{R} \rightarrow \mathbb{R}$ such that

- a) $f(A)$ is not open for some open set A .
- b) $g(B)$ is not closed for some closed set B .

3. p.139 # 13