MA 101, Spring 2007

Solutions to the collected set # 1.

Problem 1 a. *The skies are not cloudy all day.*

Statement with quantifiers: For every place on Earth, at every day there is a time when the skies are not cloudy.

Negation: There exists a place and a day such that the skies are cloudy all this day at this place.

Recasted negation: At some place and time it is cloudy all day.

Remark: Other interpretations are possible. For example, I included *place* which was not explicitly in the saying. This illustrates ambiguity of our everyday language.

Problem 2 d. *Two points in the plane determine a line.*

Statement with quantifiers: For every two points x, y in the plane, such that $x \neq y$ there exists a unique line L containing both points (i.e. such that $x \in L$ and $y \in L$)

If one clarifies what does unique mean, he/she will get a more detailed statement

A more detailed statement: For every two points x, y in the plane, such that $x \neq y$ there exists a line L containing both points, and, moreover, any line L_1 containing both points ($x, y \in L_1$) coincides with L ($L_1 = L$).

As we can see, this statement, consists of 2 simpler ones: existence of L and its uniqueness. The equivalent reformulation below separates existence from uniqueness;

Another version: For every two points x, y in the plane, such that $x \neq y$ there exists a line L containing both points (existence). Moreover (the logical meaning here is **and**), for every two points x, y in the plain, such that $x \neq y$ and for every two lines L_1 and L_2, the conditions $x, y \in L_1$ and $x, y \in L_2$ imply that $L_1 = L_2$

Negation: There exist two points $x \neq y$ on the plane such that either there is no line containing both of them (every line in the plane does not contain both points) or there are 2 lines $L_1 \neq L_2$, each containing both points.

Recasted negation: There exist 2 points $x \neq y$ which do not determine a line

Alternative recasting: 2 (different) points do not always determine a line.

Problem 4, s 1.2

Let X be an uncountable set, and let $C \subset X$ be a countable one. We can write

$$X = (X \setminus C) \cup C.$$

If we assume that $X \setminus C$ is countable, then X must be countable, as a union of 2 countable sets. So the set $X \setminus C$ is uncountable.