Symbols for the differential:

$df_x h$

$df(x)h$

$df(x;h)$

$f'(x)h$

if $f : \underset{\mathbb{R}^n}{\overset{\cap}{\Omega}} \to \mathbb{R}$ then $f' = (f_{x_1}, f_{x_2}, \dots, f_{x_n})$

$$\nabla f := (f')^T = \begin{pmatrix} f_{x_1} \\ f_{x_2} \\ \vdots \\ f_{x_n} \end{pmatrix}$$

<u>Thm</u> Let $f : \underset{\mathbb{R}^n}{\overset{\cap}{\Omega}} \to \mathbb{R}$ be st $\frac{\partial f}{\partial x_k}$ exist in a nbhd of $\vec{x}_0 \in \Omega$ and $\frac{\partial f}{\partial x_k}$ are continuous at $\vec{x}_0$. Then f is differentiable at $\vec{x}_0$

(Remark: The existence of all partial derivatives is not sufficient for differentiability)

<u>Pf</u>: $\vec{h} := (h_1, h_2, \dots, h_n)^T$

define $x_0 = x$

$x_1 = x_0 + h_1 e_1$

$x_2 = x_1 + h_2 e_2$

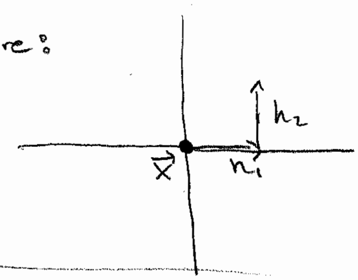
$\vdots$

$x_{k+1} = x_k + h_{k+1} e_{k+1}$

Then $f(\vec{x}+\vec{h}) - f(\vec{x}) = \sum f(\vec{x}_k) - f(\vec{x}_{k-1}) =$ (use MVT)

$\sum D_k f(\vec{x}_{k-1} + \theta_k h_k \vec{e}_k) h_k \quad (\theta \in (0,1))$

Picture:



Mean Value theorem:

$f(b) - f(a) = f'(x)(b-a)$

$x \in (a,b)$

Now let $L := \left(D_1 f(\vec{x}_0),\ D_2 f(\vec{x}_0),\ \ldots,\ D_n f(\vec{x}_0)\right)$ (the matrix of partial derivatives)

$$\text{Then } f(\vec{x}_0 + \vec{h}) - f(\vec{x}_0) - L\vec{h} = \sum_{k=1}^{n} \underbrace{\left[D_k f(x_k + \Theta_k h_k \vec{e}) - D_k(x_0)\right]}_{\text{This bracketed difference tends to } 0 \text{ as } \|\vec{h}\| \to 0} h$$

So the whole sum $= o(h)$ $\left(r(\vec{h}) = o(\vec{h}) \text{ iff } \lim_{\|\vec{h}\| \to 0} \frac{r(\vec{h})}{\|\vec{h}\|} = 0 \right)$