Homework assignment, September 19, 2007.

1. Let X, Y be metric spaces, and let $f : X \to Y$. Write the definition of uniform continuity of f. What does it mean that f is *not* uniformly continuous? (Write an ε–δ statement).

2. Prove that the space of all bounded functions $f : X \to Y$ (where (Y, d) is a complete metric space and X is some set) with the metric $\rho(f, g) = \sup\{d(f(x), g(x)) : x \in X\}$ is a complete metric space.

3. Prove that the space of continuous real-valued functions on the interval $[0, 1]$ with the metric $\rho(f, g) = \|f - g\|_2$, where $\|f\|_2^2 = \int_0^1 |f(x)|^2 dx$ is not complete. **Hint:** try to construct a sequence whose limit is not continuous.

4. Let X be a metric space and let $a \in X$. Show that for any $r > 0$ the *open ball* $\{x \in X : \rho(x, a) < r\}$ is indeed an open set, and the closed ball $\{x \in X : \rho(x, a) \leq r\}$ is indeed closed.