Homework assignment, September 26, 2007.

1. Let $X = [0,1) \cup [2,3] \cup (4,5)$ with relative topology (inherited from $\mathbb{R}$). For each of the subsets of X below indicate whether it is closed, open, both closed and open, or neither closed nor open (in the relative topology): $[0,1)$, $(1/2,1)$, $(2,2.5)$, $[2,3)$, $(4,4.5]$, $(4,4.5)$, $(4,5)$, $(2.5,3] \cup (4,4.5)$, $[2.5,3] \cup (4,4.5]$.

Justify your conclusions.

2. Give an example of a continuous bijection $f : X \to Y$ such that f^{-1} is not continuous. Be sure to explain the details.

3. For a non-empty subset A of a metric space define the distance $d(x,A)$ from a point x to the set A as $d(x,A) := \inf\{\rho(x,y) : y \in A\}$.

Show that $d(x,A) = 0$ iff $x \in \operatorname{clos} A$;

4. Show that the function $x \mapsto d(x,A)$ introduced in the previous problem is a continuous function on X.