Homework assignment, October 1, 2007.

1. Let X be a connected subset of $\mathbb{R}^n$ and let $f : X \to \mathbb{R}^m$ be a continuous function. Show that the graph Γ_f of f

$$\Gamma_f := \{(x, f(x)) \in \mathbb{R}^{n+m} : x \in X\}$$

is a connected subset of $\mathbb{R}^{n+m}$.

2. Let X be a metric space, Y be a topological space and let $x_0 \in X$ be an accumulation point of a subset $X_0 \subset X$. Let $f : X_0 \to Y$. Prove that $y_0 = \lim\limits_{x \to x_0} f(x)$ if and only if for every sequence $\{x_n\}_{n=1}^{\infty}$ in X such that $x_n \neq x_0 \; \forall n \in \mathbb{N}$ and $\lim\limits_{n \to \infty} x_n = x_0$ we have $\lim\limits_{n \to \infty} f(x_n) = y_0$.

Note, that in one direction it is sufficient to assume that X is only a topological space.

3. Using the definition find the following limits (if exist)

a) $\displaystyle \lim_{(x,y) \to (0,0)} \frac{x^4 + y^4}{x^2 + y^2}$;

b) $\displaystyle \lim_{(x,y) \to (0,0)} \frac{xy^2}{x^2 + y^2}$;