Homework assignment, Nov. 5, 2007.

1. Find

$$\lim_{n \to \infty} \lim_{m \to \infty} (1 - \cos^m(n!2\pi x))$$

2. Let A be a square matrix, ||A|| < 1 (the operator norm). Prove that

$$(I - A)^{-1} = \sum_{k=0}^{\infty} A^k$$

(here we assume that $A^0 = I$).

3. Apply the contractive mapping principle to the affine map ϕ in \mathbb{R}^N , $\phi(\mathbf{x}) = A\mathbf{x} + \mathbf{b}$, where A is a square $N \times N$ matrix, ||A|| < 1. Show that if you pick the initial value $\mathbf{x}_0 = \mathbf{b}$, you get the formula for $(I - A)^{-1}$ from the previous problem.

What happen if you chose an arbitrary initial value \mathbf{x}_0 ?

4. Let \mathcal{A} be the collection of sets in \mathbb{R} that can be represented as a finite union of disjoint intervals of form $(a_k, b_k]$ (intervals of form $(-\infty, b]$ and (a, ∞) are also allowed).

Show that \mathcal{A} is the algebra generated by all intervals (a, b], i.e. that \mathcal{A} is the smallest algebra of sets containing all the intervals (a, b].