Homework assignment, Nov. 12, 2007.

1. Let μ_n be an increasing sequence of outer measures on X (i.e. for any $A \subset X$ we have $\mu_n(A) \leq \mu_{n+1}(A)$, $n = 1, 2, 3, \ldots$), and let $\mu(A) = \lim_{n\to\infty} \mu_m(A)$.

   Prove that μ is also an outer measure.

2. Let μ_0 be a premeasure defined on an algebra $\mathcal{A}$, and let μ^* be the outer measure constructed from μ_0, and let $\mathcal{M}$ be the set of all μ^*-measurable sets.

   Prove that if $B \in \mathcal{M}$, $\mu^*(A) < \infty$, then given any $\varepsilon > 0$ there exists a set $A = \cup_1^\infty A_k$, $A_k \in A$ such that $B \subset A$ and $\mu^*(A \setminus B) < \varepsilon$.

   Hint: consider a cover of B by A_k which almost gives $\mu^*(B)$.

3. Conclude from the previous problem, that if $B \in \mathcal{M}$, $\mu^*(B) < \infty$ then given $\varepsilon > 0$ there exists $A \in \mathcal{A}$ such that $\mu^*(B \Delta A) < \varepsilon$. All the notation is the same as in the previous problem.