Homework assignment, Dec. 3, 2007.

1. Let μ_0 be a premeasure defined on an algebra $\mathcal{A}$, and let μ^* be the outer measure constructed from μ_0, and let $\mathcal{M}$ be the set of all μ^*-measurable sets.

Prove that if $B \in \mathcal{M}$, then given any $\varepsilon > 0$ there exists a set $A = \cup_1^N A_k$, $A_k \in A$ such that $B \subset A$ and $\mu^*(A \setminus B) < \varepsilon$.

Hint: consider a cover of B by A_k, $k \in \mathbb{N}$ which almost gives $\mu^*(B)$.

2. Let $f \geq 0$ be such that

$$\mu\{x : f(x) > t\} = \frac{1}{1+t^2}$$

Compute $\int f d\mu$.

3. Give an example of simple functions f_n, f, $f = \lim f_n$ such that

$$\int f d\mu \neq \lim_{n\to\infty} \int f_n d\mu.$$