Math 18 Intermediate Calculus
EXAM 2, November 8, 2006

NAME: **Section #:**
I.D.: **Instructor:**

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Write your name and I.D. number on every page you use. Write your answers clearly. Use words if you feel you need to explain your work. No books or notes allowed during the exam. Calculators not allowed (and not necessary). All your work should be on these pages or backs. Partial credit on incomplete or erroneous answers can only be given if you provide explanation of your work along with the answer.

(1) (a) Find the critical points of the function

$$f(x, y) = x^2 - xy^2 - 4x.$$

(b) Describe what the Second Derivative test says about each critical point.

ANSWERS:_____

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(2) Consider the function $f(x, y) = \cos x + y^2 \sin x$ and the point

$$(x_0, y_0) = (0, 3).$$

(a) Find the gradient $\nabla f(x_0, y_0)$

(b) Write the linear approximation formula for $f(x, y)$ near $f(x_0, y_0)$, and write the approximate value it gives for $f(0.1, 2.9)$.

(c) For an arbitrary unit vector $\mathbf{u} = (a, b)$, write a formula for the directional derivative $D_{\mathbf{u}} f(x_0, y_0)$.

(d) Find the equation of the tangent **line** to the **level curve** $f(x, y) =$ constant, at the point (x_0, y_0).

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(3) Consider the function $f(x, y) = x^2 + y^2 + x$ and the region $D = \{(x, y)|x^2 + y^2 \leq 3\}$

(a) Set up an integration to find the volume below the graph of f over the region D using rectangular coordinates. Be sure to set up the bounds, but do not evaluate the integral.

(b) Set up an integration to find the volume below the graph of f over the region D using polar coordinates. Be sure to set up the bounds, but do not evaluate the integral.

(c) Evaluate one of the above integrals (your choice).

ANSWER:_____

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(4) (a) The mass density of a space solid E is given by the function $\sigma(x, y, z)$. Give a fomula for the total mass as an integral over the region E.

(b) Consider the space solid E bounded below by the cone $z = \sqrt{x^2 + y^2}$ and above by the top of the sphere

$$x^2 + y^2 + (z - 1)^2 = 1.$$

Given that the mass density is

$$\sigma(x, y, z) \quad = \quad z \cdot \sqrt{x^2 + y^2 + z^2},$$

calculate the total mass of the solid. Choose your coordinates appropriately!

ANSWER:_____

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(5) Consider the region D defined by

$$1 \leq x + y \leq 2$$
$$0 \leq x - 2y \leq 2.$$

and the integral

$$I = \iint_D (x + y)(x - 2y)^2 \, dA.$$

Also write $u = x + y$ and $v = x - 2y$.

(a) Find the Jacobian $\dfrac{\partial(x, y)}{\partial(u, v)}$.

(b) Rewrite the integral I above in terms of the coordinates u, v.

(c) Evaluate the integral.