Homework assignment, Oct. 8, 2008.

Discussed next class

1. Let A be the multiplication by the independent variable x operator in $L^2(0,1)$, $Af(x) = xf(x)$. Show that the range of this operator is not closed. Recall that $\operatorname{Ran} A = \{Af : f \in D(A)\}$

Hint: Consider, for example functions $g_n = \chi_{[1/n,1)}$.

2. Prove the theorem about ranges and kernels of the operators

a) $\operatorname{Ker} A^* = (\operatorname{Ran} A)^\perp$

b) $\operatorname{Ker} A = (\operatorname{Ran} A^*)^\perp$

c) $\operatorname{clos} \operatorname{Ran} A = (\operatorname{Ker} A^*)^\perp$

d) $\operatorname{clos} \operatorname{Ran} A^* = (\operatorname{Ker} A)^\perp$