Homework assignment, Sept. 22, 2008.

To be collected Friday, Sept. 26.

1. Apply Gram–Schmidt orthogonalization to the system of functions $1, x, x^2, x^3, \dots$ in the space $L^2(-1, 1)$. You will get some classical *orthogonal polynomials* there.

2. Show that in a complex Hilbert space $\mathcal{H}$ the following identities are satisfied

a) $(x, y) = \dfrac{1}{N} \displaystyle\sum_{k=0}^{N-1} e^{2\pi i k/N} \|x + e^{2\pi i k/N} y\|^2$ for $N \geq 3$.

b) $(x, y) = \dfrac{1}{2\pi} \displaystyle\int_0^{2\pi} e^{it} \|x + e^{it} y\|^2 dt$.

3. s. 2.4, # 9, 13