

MATH 2250, Fall 2010.

Homework assignment, Oct. 6, 2010

Will be collected Friday, Oct. 8.

1. Describe all linear fractional transformations that carry circles $|z| = r$ and $|z| = R$ ($r \neq R$) into concentric circles (not necessarily centered at 0).
2. Prove that a conformal automorphism of the upper half plane $\text{Im } z > 0$ (i.e. a conformal map from the upper half plane onto itself) is a linear fractional transformation. Describe all conformal automorphisms of the upper half-plane.
3. p. 83 # 7 (was in one of the previous assignments).
4. Find the image of the unit disc under the map $f(z) = z + 1/z$. Show that this map is conformal.
5. p. 96, # 1. **Hint:** It is easy to map the region to a angle. Using the map $z \mapsto z^a$ one then can transform the angle to a half-plane, and the rest is easy. You just need to keep track of where the axis of symmetry go.
6. Find the image of the disc $|z - 2i| < 2$ under the transformation

$$f(z) = \frac{z - 1}{z + 2}.$$

(was in the previous assignment)