MATH 2250, Fall 2010.
Homework assignment, Oct. 15, 2010

1. Let f be an analytic in the unit disc $\mathbb{D}$ function, $|f(z)| \leq 1$ for all $z \in \mathbb{D}$, and let

$$f = \sum_{n \geq 0} a_n z^n$$

be its Taylor series.

Prove that $|a_n| \leq 1$ for all n.

2. Let f_k be analytic functions in the unit disc D such that

$$f_k(z) = \sum_{n \geq 0} a_n(k) z^k, \qquad |a_n(k)| \leq 1.$$

Assume that for all n

$$\lim_{k \to \infty} a_n(k) = a_n.$$

Show that f_n converges (uniformly on compact subsets of D) to

$$f(z) = \sum_{n \geq 0} a_n z^n.$$

3. Combining two above problems, show that for any bounded sequence of functions f_n ($\|f_n\|_\infty \leq M$) analytic in the unit disc, one can find a convergent (uniformly on the compact subsets) subsequence.

4. p. 178, # 1, 2.