MATH 2250, Fall 2010.
Homework assignment, Oct. 22, 2010

1. Let Ω be a simply connected region in $\mathbb{C}$ and let $z_0 \in \Omega$. Let $\mathcal{F}^1$ be the set of all functions f holomorphic in Ω and such that

a) $|f(z)| \leq 1$ for all $z \in \Omega$;

b) $f(z_0) = 0$, $f'(z_0) > 0$.

Can you describe the function for which $f'(z_0)$ is maximal?

2. Compute the Laplacians of the function

$$\frac{1 - |z|^2}{|1 - \overline{z}\xi|^2}, \qquad |z| < 1, \ |\xi| \leq 1$$

in variables ξ and z.

3. Let $a, z \in \mathbb{C}$, $|a| < 1$ and $|z| \leq 1$. Compute

$$\sum_{n \geq 0} a^n \overline{z}^n + \sum_{n > 0} \overline{a}^n z^n$$