# MATH 2250, Fall 2010.
# Homework assignment, Nov. 19, 2010

**Will be collected Monday, Nov. 29.**

1. Let $U \subset \mathbb{C}$ be a connected open set, and let $a \in U$. Show that any rational function with poles in $U$ can be approximated uniformly in the complement of $U$ by rational functions with the only pole at $a$.

**Hint:** Show that if $z_1$ is close to $z_0 \in U$, then a rational function with a pole at $z_0$ can be uniformly in the complement of $U$ approximated by rational functions with a pole at $z_1$. Use a continuous induction to move $z_1$ from a neighborhood of $z_0$ to all of $U$.

2. Show that the function $f(z) = \overline{z}$ cannot be uniformly on the unit circle $\mathbb{T}$ approximated by polynomials.

3. Let $\Omega = \{z \in \mathbb{C} : 0 < |z| < 1\}$, and let $h$ be a function on $\partial\Omega$ defined by

$$h(z) = \begin{cases} 0 & |z| = 1; \\ 1 & z = 0 \end{cases}$$

Let $\mathcal{F} = \mathcal{F}(h)$ be the collection of all subharmonic in $\Omega$ functions $v$, such that for all $\xi \in \partial\Omega$

$$\limsup_{z \to \xi} v(z) \le h(z).$$

Compute the function $u$ on $\Omega$,

$$u(z) = \sup\{v(z) : v \in \mathcal{F}\}, \qquad z \in \Omega.$$

In other word, find what the Perron process gives for the boundary data $h$.

4. Compute the harmonic extension of a function $h$ defined on the unit circle by $h(z) = |1-z|^2$ to the unit disc. Write the Poisson formula and compute the integral similarly to the integral in the midterm

Read s.1, 2, 3 of Ch. 5.

5. p. 178 #1, p. 184 # 5, p. 193 # 2.