Riemann Surfaces (Continued)

$\mathcal{G}(\mathbb{C})$, Riemann surface of $\mathbb{F}_\xi$ is a connected component of $\mathcal{G}(\mathbb{C})$ containing $\mathbb{F}_\xi$.

Each point of $\mathcal{G}(\mathbb{C})$ has a neigh "biholomorphic" to $\mathbb{D}$.

This is clear, for $\mathbb{F}_\xi$ we can choose (f, U, ξ) can shrink to get disc. We know for locally euclidean spaces, connected is equivalent to path connected. So R.S is connected, by our definition, and so path connected. Analytic continuation along a path γ in the Rieman surface, How?

$\pi : \mathcal{G}(\mathbb{C}) \longrightarrow \mathbb{C}$, $\pi(\mathbb{F}_\xi) = \xi$, called projection. Now if $\gamma : [a,b] \longrightarrow \mathbb{C}$, $\gamma(a) = \xi$, $\gamma(b) = \eta$ and $\mathbb{F}_\xi$ are given. <u>Analytic Continuation of $\mathbb{F}_\xi$</u> is $\tilde{\gamma} : [a,b] \longrightarrow$ R.S at $\mathbb{F}_\xi$, s.t. $\pi \tilde{\gamma}(t) = \gamma(t)$.

(also, $\pi(a) = \mathbb{F}_\xi$).

Analytic Continuation along a path γ is unique: If

$\exists\, \tilde{\gamma}, \tilde{\tilde{\gamma}} : [a,b] \longrightarrow \mathcal{G}(\mathbb{C})$ s.t. $\tilde{\gamma}(a) = \tilde{\tilde{\gamma}}(a) = \mathbb{F}_\xi$, $\pi \circ \tilde{\gamma} = \pi \circ \tilde{\tilde{\gamma}} = \gamma$

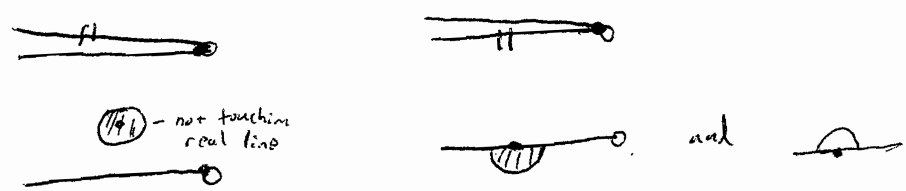
$\Rightarrow \tilde{\gamma} = \tilde{\tilde{\gamma}}$.

$\forall \mathbb{F}_\xi$ $\exists$ nbhd U of $\mathbb{F}_\xi$ st $\Pi|_U$ is bijection, which will force agreement on overlaps.

<u>Monodromy Thm:</u> Let $\gamma : [a,b] \times [0,1] \longrightarrow \mathbb{C}$, (continuous family of paths, ~~and for any $s \in [0,1]$ there exists an analytic~~ $\gamma(a,s) = \xi$, $\gamma(b,s) = \zeta$; and for all s. there is an analytic continuation of $\mathbb{F}_\xi$ along $\gamma_s(\cdot) = \gamma(\cdot, s)$, then the analytic continuation at ζ along all γ_s coincide.

(Idea: show for s_1, s_2 close enough, we have agreement (use uniqueness result) then exploit compactness of $[a,b]$.)

$\sqrt{z}$:



point not on real line

point on real line